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MATHEMATICS 7

Patterns and Relations

Distance Learning

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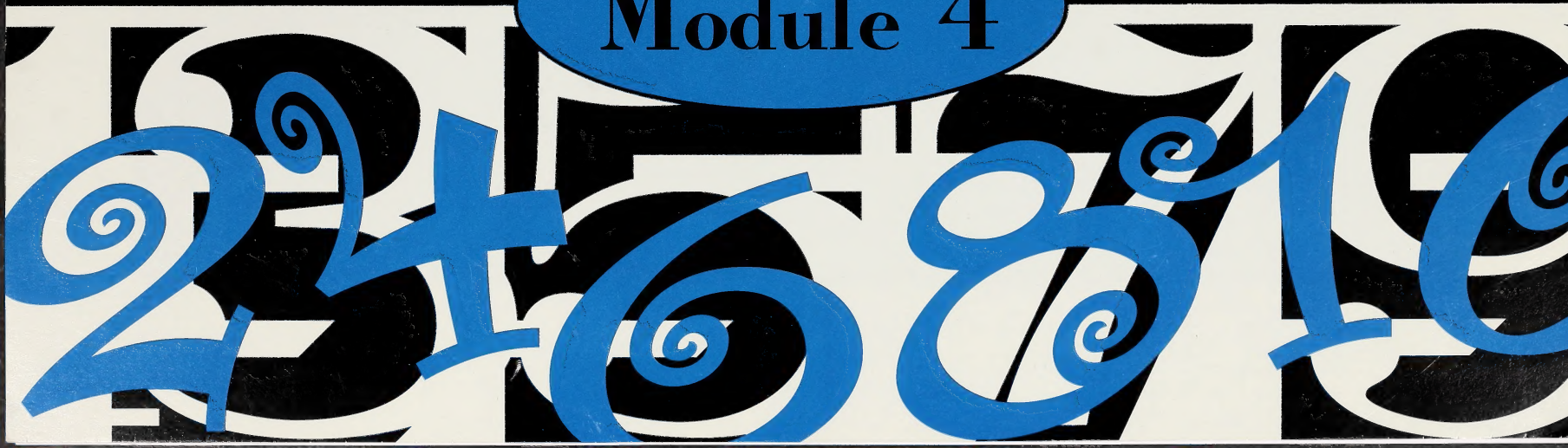
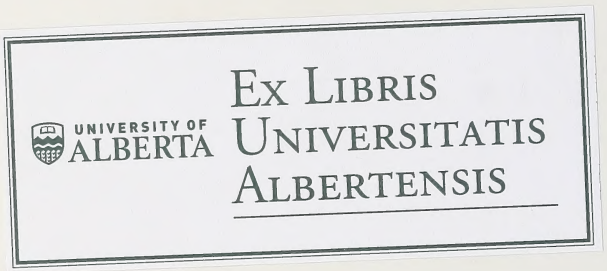
Module 4

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Module 4

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Module 4
Patterns and Relations
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Teachers (Mathematics 7)	✓
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Welcome

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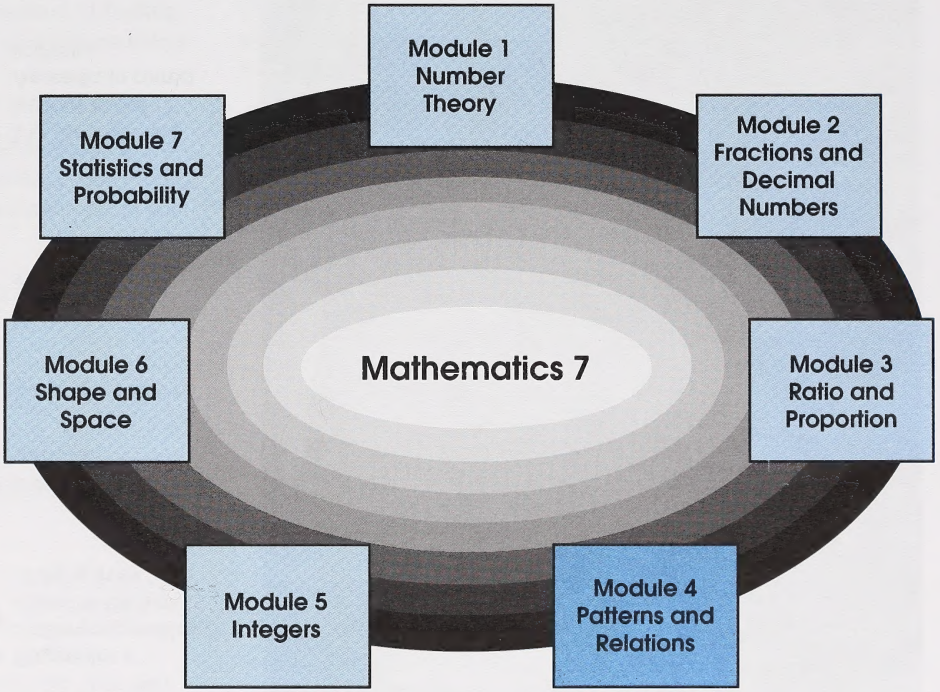
Welcome to Module 4. We hope you'll enjoy your study of Patterns and Relations.

Mathematics 7 contains seven modules. Work through the modules in the order given, since several concepts build on each other as you progress in the course.



The document you are presently reading is called a Student Module Booklet. You may find visual cues or icons throughout it. Read the following explanations to discover what each icon prompts you to do.

- Prepare for a problem that will provide a change of topic.

- Prepare for a challenging problem related to the topic of the activity.

- Use the Internet to explore a topic.

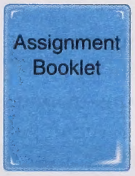
- Use computer software.

- Use a scientific calculator.

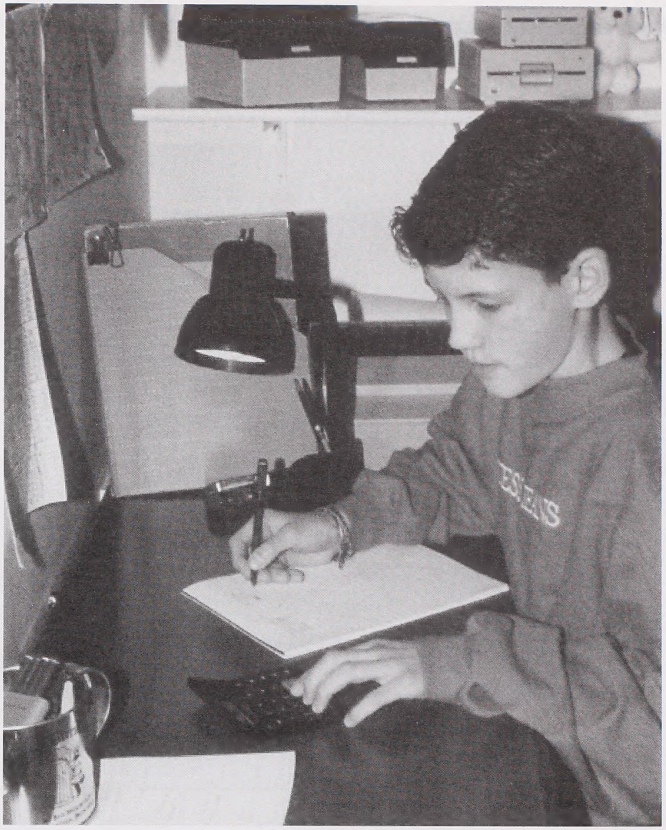
- View a videocassette.

- Pay close attention to important words or ideas.

- Use the suggested answers in the Appendix to correct activities.



- Answer the questions in the Assignment Booklet.



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There are no response spaces provided in this Student Module Booklet. This means that you will need to use your own paper for your responses. You should keep your response pages in a binder so that you can refer to them when you are reviewing or studying.

Problem-Solving Skills

One of the exciting features of this course is that you will develop and improve your ability in problem solving. You will need these problem-solving skills many times in your lifetime. Since this course focuses on problem solving, it is important that you understand what a **problem** is.

A problem is a task for which the method of finding the answer (as well as the answer) is not immediately known.

Like any skill, the skill of problem solving must be developed. Problems may or may not involve computation (adding, subtracting, multiplying, and dividing). Some problems are realistic; others are puzzles.

You will have the opportunity in most activities to try a problem-solving challenge.

The icon is a cue that the problem will be related to the topic of the activity.

The icon is a cue that the problem will provide a change of topic.

The Four-stage Process

There are four stages that can be used to solve any problem: understanding the problem, developing a plan, trying the plan, and looking back.

Understanding the Problem

In this stage you should expect to feel puzzled. There are various reasons for feeling this way.

- You may not know the meanings of all the words.
- You may not understand the situation in the problem.
- You may be confused by unnecessary information.

Once you understand the problem, you should think about the problem and make an estimate of what the answer should be. This will help you arrive at a reasonable answer.

Developing a Plan

This is where you should decide on the plan of action that you are going to take to solve the problem.

You may consider the following strategies:

- changing your point of view
- using objects
- using diagrams
- making an organized list
- using Venn diagrams
- making a table
- guessing, checking, and revising
- acting out a problem
- working backwards
- simplifying a problem
- finding and applying a pattern
- using elimination
- using truth tables
- using an equation

Note: The Appendix in Module 1 explains these strategies in detail. When you see a problem-solving icon in any module, you should turn to the Appendix in Module 1 and review the problem-solving strategies.

Trying the Plan

In this stage you should try the plan and see if it works.

Be sure to work carefully and record your progress. You are encouraged to use a calculator to help with your calculations.

Note: While trying the plan, you should monitor your progress in order to determine if your plan will lead to a solution. You may find that the plan will not produce a solution, in which case a new plan will have to be developed.

Looking Back

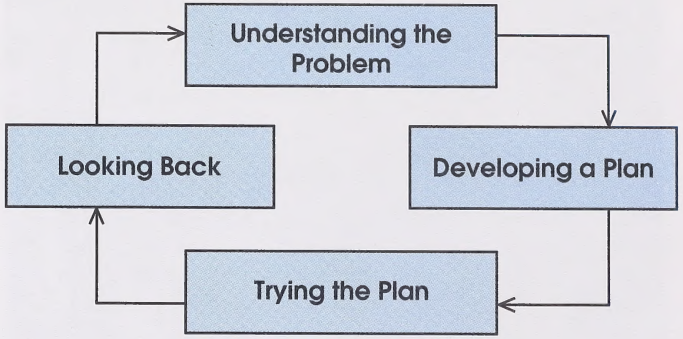
In this stage you should look back at the problem and compare your answer to the estimate you made in the first stage. Restate the problem using your answer.

Ask yourself these questions: "Did my plan work? Is my answer reasonable?"

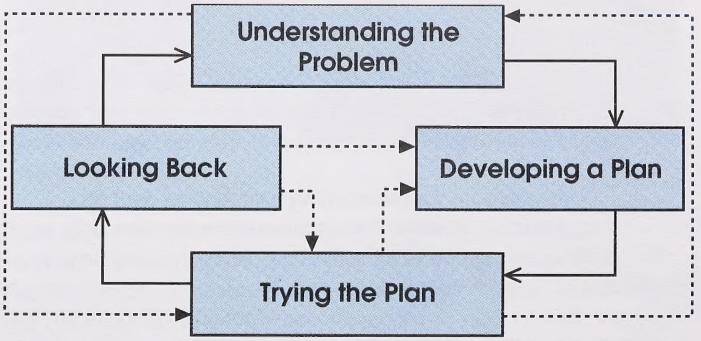
If you did not arrive at an answer, another strategy may work better. If your answer is unreasonable, you may have made errors while trying your plan.

Sequence of Stages

You usually approach a problem in the order outlined in the following diagram.



If you encounter difficulties in your original plan, or if you realize that another strategy will have better results, you may need to return to an earlier stage or use the stages in a different sequence.



CONTENTS

Module Overview 1
Evaluation 2

Section 1: From Arithmetic to Algebra 3
Activity 1: Mathematical Expressions 4
Activity 2: Variable Expressions 13
Activity 3: Equations 19
Follow-up Activities 25
Extra Help 26
Enrichment 28
Conclusion 32
Assignment 32

Section 2: Patterns and Relations 33
Activity 1: Describing Patterns and Relations 34
Activity 2: Graphs of Relations 44
Follow-up Activities 54
Extra Help 54
Enrichment 57
Conclusion 59
Assignment 59

Section 3: From Measurement to Algebra 60
Activity 1: Perimeter and Area of a Rectangle 61
Activity 2: Volume of a Rectangular Prism 70
Activity 3: Circumference of a Circle 73
Follow-up Activities 77
Extra Help 77
Enrichment 81
Conclusion 84
Assignment 84

Module Summary 85
Final Module Assignment 85

Appendix 86
Glossary 87
Suggested Answers 88
Cut-out Learning Aids 131

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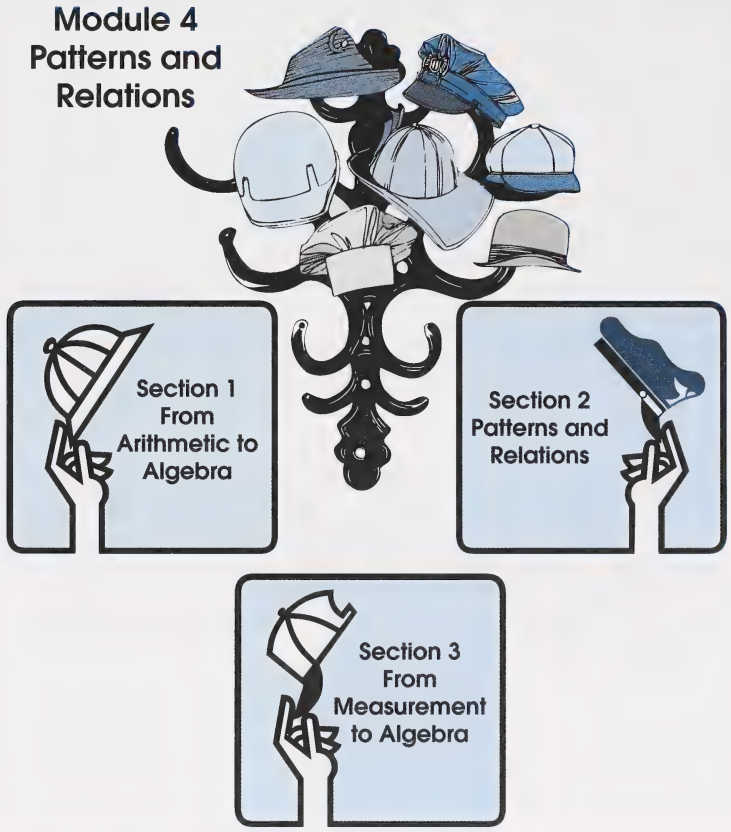
Module Overview

What occupation do you think you will pursue when you complete school? What will you be doing in your chosen occupation?

Did you know that many jobs require you to use mathematics? In fact, almost everybody uses mathematics in their daily lives. Mathematics is necessary for counting, measuring, and designing. It enables people to keep records, analyse patterns, and make decisions. It provides a language for communicating.

In this module you will explore a branch of mathematics called algebra. You will discover how algebra is related to arithmetic. You will use algebra to describe patterns and to develop formulas. You will discover how algebra can be used to calculate the perimeter and area of a rectangle, the volume of a rectangular prism, and the circumference of a circle.



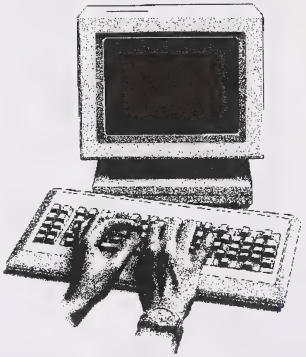
Evaluation

Your mark for this module will be determined by how well you complete the assignments at the end of each section and at the end of the module. In this module you must complete four assignments. The mark distribution is as follows:

Section 1 Assignment	**30 marks**
Section 2 Assignment	**25 marks**
Section 3 Assignment	**25 marks**
Final Module Assignment	**20 marks**
TOTAL	**100 marks**

When doing the assignments, work slowly and carefully. You must do each assignment independently, but if you are having difficulties, you may review the appropriate section in this module booklet.

If you are working on a CML terminal, you will have a module test as well as a module assignment.



Note

There is a final supervised test at the end of this course. Your mark for the course will be determined by how well you do on the module assignments and the supervised final test.

Section 1: From Arithmetic to Algebra

Birds and animals are often used to represent ideas. They become symbols. For example, the beaver is a symbol of Canada, an ant represents hard work, and a turtle stands for slowness. What does an owl represent?

Symbols are also used in mathematics. For example, "2" is the symbol for "two," "+" is the symbol for "add," and "×" is the symbol for "multiply."

In this section you will discover that letters can be used as symbols to represent numbers that are not given. You will evaluate mathematical expressions and variable expressions. You will translate English phrases into mathematical and variable expressions and you will write equations. Finally, you will solve equations by inspection and by guessing, checking, and revising your guesses.

Activity 1: Mathematical Expressions

Evaluating Mathematical Expressions

Have you ever entered a contest to win a trip, a bicycle, or some other prize?

Often the winner of a contest is asked a skill-testing question that involves mathematics. Here is an example.

The contest winner must first correctly answer this skill-testing question:

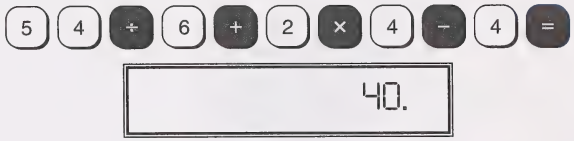
Find the value of $54 \div 6 + 2 \times 4 - 4$.

The phrase $54 \div 6 + 2 \times 4 - 4$ is a **mathematical expression**.

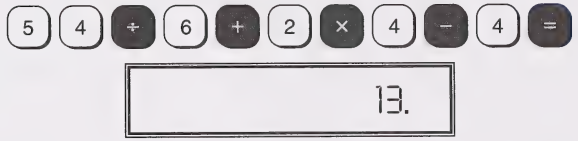
A mathematical expression is a phrase that involves numbers and operations.

What is the value of the mathematical expression $54 \div 6 + 2 \times 4 - 4$?

If you use an ordinary calculator to find the value, you will obtain this result.



If you use a scientific calculator to find the value, you will obtain this result.



Why is there a different result?

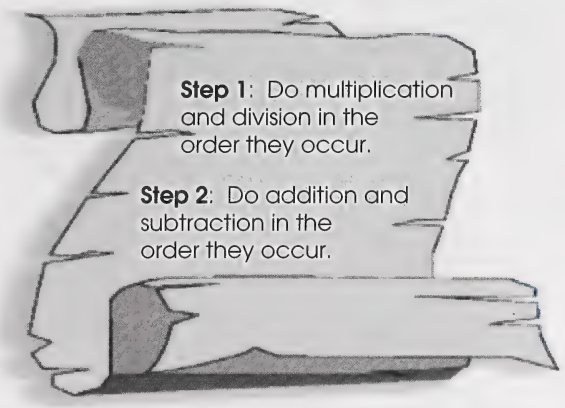
There is a different result because the scientific calculator uses the rules for the **order of operations** and the ordinary calculator does not.

The order of operations is a set of rules that mathematicians have agreed upon to find the value of a mathematical expression.

Another word for finding the value of a mathematical expression is **evaluating**.

When you use a paper-and-pencil method to evaluate an expression with two or more operations, you use the following rules.

Rules for the Order of Operations



Example 1

Find the value of the mathematical expression $54 \div 6 + 2 \times 4 - 4$.

Solution

$54 \div 6 + 2 \times 4 - 4$ ← Do multiplication and division in the order they occur.

$= 9 + 8 - 4$

$= 17 - 4$ ← Do addition and subtraction in the order they occur.

$= 13$

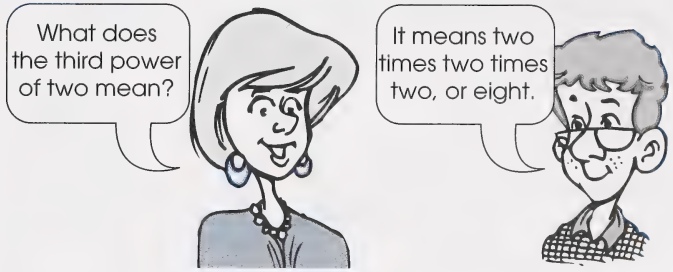
1. Use the paper-and-pencil method to evaluate each of the following mathematical expressions.

a.	$4+5\times3$	**b.**	$9-4\times2$	**c.**	$8\div4+2$
d.	$10-3\div3$	**e.**	$8\times3-2\times4$	**f.**	$6+5\times7+4$
g.	$9\div3\times2+8$	**h.**	$2\times8-3\times4$	**i.**	$9\times6+3+5\times6$

Check your answers by turning to the Appendix.

Mathematical expressions may contain **powers**. You found out about powers in Module 1.

A power is a compact way to write a product of equal factors.



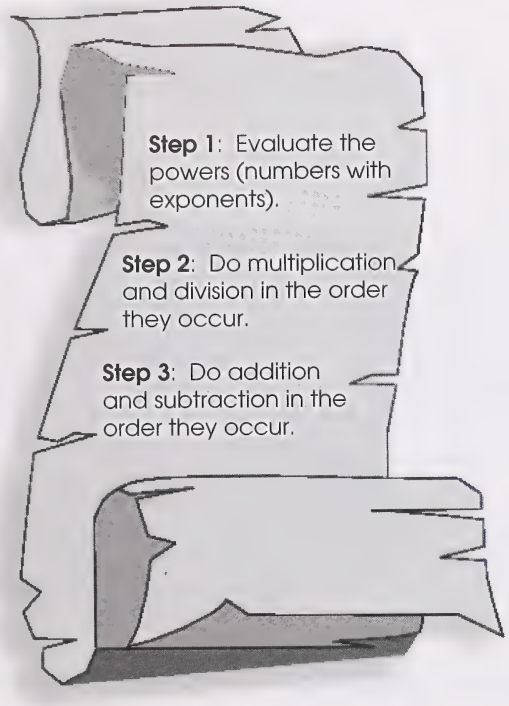
$2^3 = 2 \times 2 \times 2$

$= 8$

When an expression contains a power, you evaluate the power before you multiply or divide. So, the rules for the order of operations must be extended.

When you evaluate an expression with powers and one or more operations, you use the following rules.

Rules for the Order of Operations


Step 1: Evaluate the powers (numbers with exponents).
Step 2: Do multiplication and division in the order they occur.
Step 3: Do addition and subtraction in the order they occur.

Example 2

Find the value of the mathematical expression $2^3 + 5 \times 6$.

Solution

$2^3 + 5 \times 6$

$= 2 \times 2 \times 2 + 5 \times 6$ ← Evaluate the power.

$= 8 + 5 \times 6$ ← Multiply.

$= 8 + 30$ ← Add.

$= 38$

2. Find the value of each of the following mathematical expressions.

a. $36 \div 4 - 2^2$ **b.** $8 - 3 + 2^3$

c. $3^2 + 5 \times 6$ **d.** $2^3 + 3^3 \times 2$

Check your answers by turning to the Appendix.

In spoken conversation, pauses are used to group words and show meaning; in written language, punctuation marks are used. Here is an example.

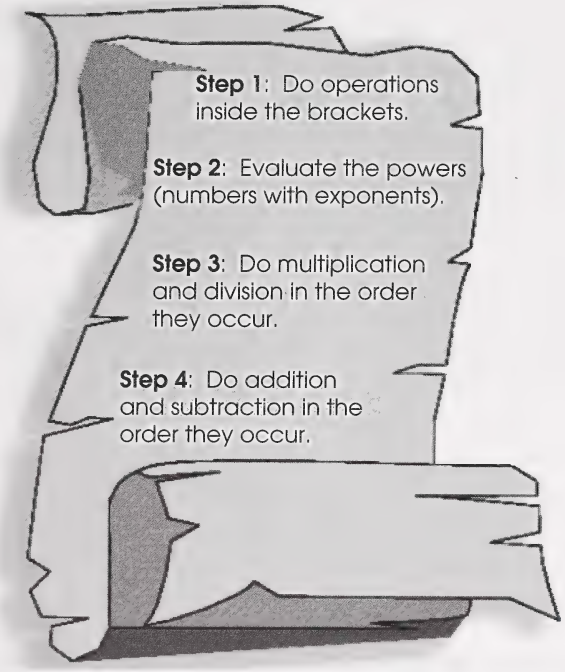
- "Mary," said Anna, "please come here."
- Mary said, "Anna, please come here."

Were you able to see a difference in meaning? In the first sentence, Anna is speaking. In the second sentence, Mary is speaking.

In mathematics, brackets are used to show groups and to indicate the order of operations. They are the "punctuation marks" of a mathematical expression.

When you evaluate a mathematical expression with brackets, you use the following rules.

Rules for the Order of Operations


Step 1: Do operations inside the brackets.
Step 2: Evaluate the powers (numbers with exponents).
Step 3: Do multiplication and division in the order they occur.
Step 4: Do addition and subtraction in the order they occur.

Example 3

Find the value of the mathematical expression $(14-6)\div 2$.

Solution

$(14-6)\div 2$ ← Perform operations inside the brackets.
$= 8\div 2$ ← Divide.
$= 4$

Example 4

Evaluate the mathematical expression $4+2\times(1+2)$.

Solution

$4+2\times(1+2)$ ← Perform operations inside the brackets.
$= 4+2\times 3$ ← Multiply.
$= 4+6$ ← Add.
$= 10$

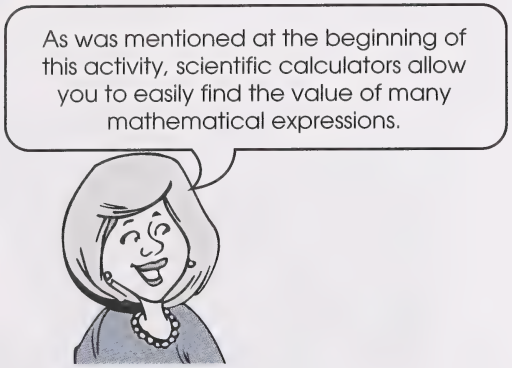
3. Use the paper-and-pencil method to find the value of each of the following mathematical expressions.

a. $(4+2)\times 7$ **b.** $(12+3)\div 3$

c. $5\times(6-2)+3$ **d.** $3^2-(4\times 2)$

e. $4^2 - 2 \times (5+3)$ f. $(18-6) \div 4$

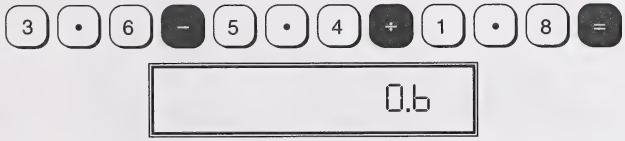
Check your answers by turning to the Appendix.


As was mentioned at the beginning of this activity, scientific calculators allow you to easily find the value of many mathematical expressions.

Example 5

Find the value of the mathematical expression $3.6 - 5.4 \div 1.8$.

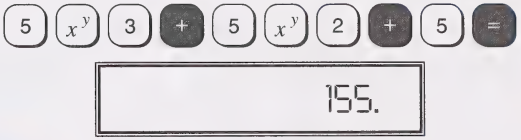
Solution



Example 6

Find the value of the mathematical expression $5^3 + 5^2 + 5$

Solution



Use a scientific calculator to answer questions 4 and 5.

4. Find the value of each of the following mathematical expressions.

a. $57 - 12 \times 3$ **b.** $7 + 0.8 \div 0.5$ **c.** $3.2 + 16.4 \div 4$

5. Find the value of each of the following mathematical expressions.

a. 2×3^2 **b.** $2 + 2^3$ **c.** $3^2 - 3 + 1$

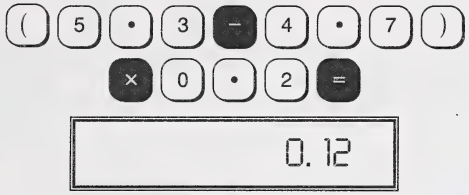
Check your answers by turning to the Appendix.

If the scientific calculator you are using has bracket keys, (and), you may use them to find the value of a mathematical expression involving brackets.

Example 7

Find the value of the mathematical expression $(5.3-4.7)\times 0.2$.

Solution



Use a scientific calculator to answer question 6.

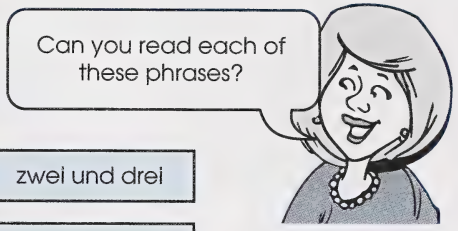
6. Find the value of each of the following expressions.

a. $42\times(91-53)$ **b.** $7.5\div(2.5-1)$

c. $(3.25+2.75)\div(5.2-4.4)$ **d.** $3.6\times(7-2)\div 3$

Check your answers by turning to the Appendix.

Translating Words and Phrases



deux et trois	zwei und drei
dos y tres	two plus three

You probably cannot read every phrase because different languages have been used, but each phrase can be translated into this mathematical expression.

$2+3$

Mathematics is a language made up of numerals and symbols which are understood by mathematicians all over the world, no matter what language is spoken.

Because non-mathematical languages such as English are very complex, a mathematical expression can be translated into many different words and phrases. For example, "$2+3$" can be translated into each of the following English phrases:

- two plus three
- two increased by three
- the sum of two and three
- three more than two
- three added to two

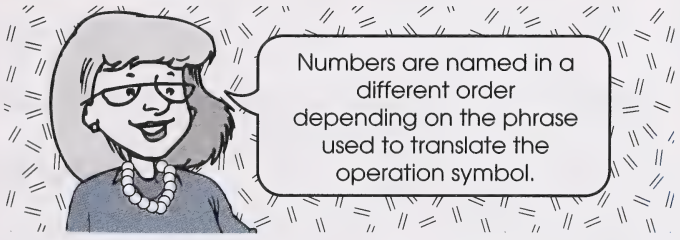
7. Translate each of the following mathematical expressions into words and phrases in as many ways as you can.

a. $6+7$ **b.** $4-1$ **c.** 2×5 **d.** $4\div3$

Check your answers by turning to the Appendix.

You have seen that mathematical operations can be expressed in many different ways.

Operation Symbol	Words and Phrases
+	plus, increased by, more than, added to, the sum of
–	minus, decreased by, diminished by, less than, subtracted from
×	times, multiplied by, the product of
÷	divided by

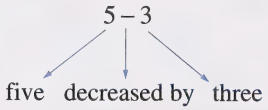


Example 1

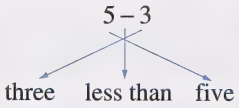
Write a phrase for the mathematical expression $5-3$.

Solution

Method 1: Using the phrase *decreased by*



Method 2: Using the phrase *less than*



8. Write mathematical expressions for each of the following phrases.

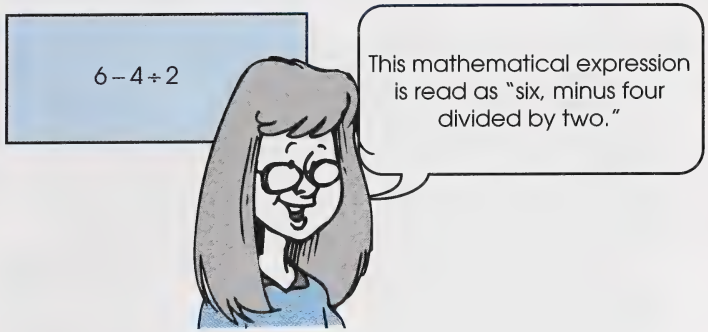
a. seven less than nine
b. six diminished by four
c. five times eight
d. two more than five
e. seven increased by six
f. nine subtracted from ten
g. twelve divided by three
h. two added to five

Check your answers by turning to the Appendix.

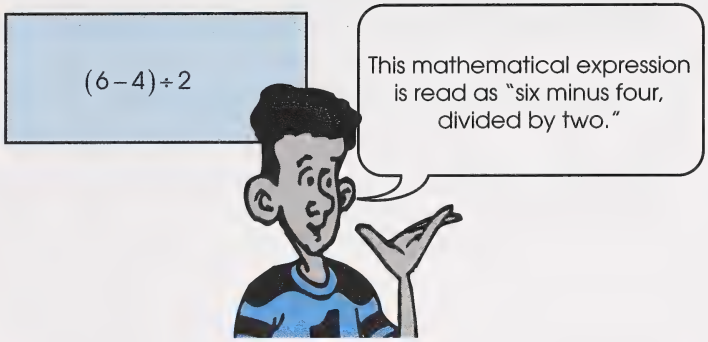
When there are two or more operations in the mathematical expression, mathematical expressions must be translated carefully so that the meaning is clear.

Now compare Examples 2 and 3.

Example 2



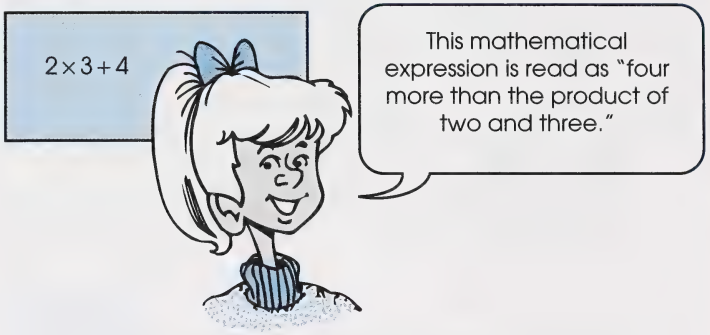
Example 3



Notice that the commas and pauses help indicate which operation is performed first.

Compare Examples 4 and 5.

Example 4



Example 5

Notice that the phrases *the product of* and *the sum of* help indicate which operation is performed first in each mathematical expression.

9. Write each of the following mathematical expressions in words. Be sure that the meaning is clear.

a.	$3\times6-5$	**b.**	$3\times(6-5)$
c.	$6-2+1$	**d.**	$6-(2+1)$
e.	$4\times5+3$	**f.**	$4\times(5+3)$
g.	$2\times6-3$	**h.**	$2\times(6-3)$
i.	$4+6\div2$	**j.**	$(4+6)\div2$

10. Write mathematical expressions for each of the following statements.

a. eight less than the product of two and three
b. twelve, minus eight divided by two
c. two more than the product of three and five
d. three times the sum of five and eight
e. five minus three, divided by two

Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following question.

11. Three people are on the bank of a river which they must cross. Their canoe can only hold 150 kg. The first person weighs 50 kg, the second person weights 75 kg, and the third person weighs 125 kg. How can all three people get across the river without sinking the canoe?

Check your answers by turning to the Appendix.

In this activity you evaluated mathematical expressions using the rules for the order of operations. You translated phrases into mathematical expressions and vice versa. You continued to solve non-routine problems.

Activity 2: Variable Expressions

Translating English Phrases

You Are In Bear Country
For your
own safety
Do not feed any wildlife
Store food inside your vehicle
Keep campsites clear of garbage

Thousands of tourists visit Canada's national parks in the mountains. The wildlife of these parks can pose a real danger to visitors. Brochures and signs warn people of the dangers of feeding animals, trying to approach animals, and driving too fast. Reading all this information can be a problem, however. To simplify the message, symbols are used. The sign in the photograph uses the symbol of a bear to caution campers.

Using symbols can simplify a written message. Symbols often have universal meanings. Octagonal signs mean stop. Inverted triangles mean yield. Arrows indicate direction.

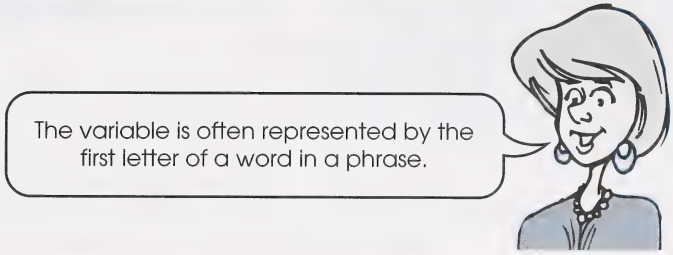
In mathematics, symbols are used to replace numbers and operations. You used symbols for numbers and operations in the previous activity. In this activity you will discover how to represent a number that is not given.

An unspecified number is called a **variable**. A mathematical expression that uses one or more variables is called a **variable expression**. Variable expressions may also be called **algebraic expressions**.

Many English phrases can be translated by using variables and variable expressions.

Example 1

Phrase	Variable
Sam's age	a
the distance from John's house to work	d
Helen's mass	m
Ruth's wages	w
the length of a house	ℓ
the width of a house	w



Example 2

Phrase	Variable Expression
five times Sam's age	$5a$
the distance from John's house to work divided by two	$\frac{d}{2}$
Helen's mass decreased by two kilograms	$m-2$
ten dollars more than Ruth's wages	$w+10$
the sum of the length and width of a house	$\ell+w$

Remember the following points when translating English phrases into variable expressions:

- Get in the habit of writing, rather than printing, variables. Be careful to make the variable x different from the multiplication operation sign $\times$. Make the variable ℓ different from the number 1. Make the variable o different from the number 0.
- Most mathematicians do not include the multiplication sign between the number and variable, but you may occasionally see brackets or the multiplication dot to separate factors. Therefore, you may see the phrase *five times Sam's age* translated as $5a$, $(5)(a)$ or $5 \bullet a$.
- Most mathematicians use a bar to show division, but you may occasionally see the division sign, $\div$, used. Therefore, you may see the phrase *the distance from John's house to work divided by two* translated as $\frac{d}{2}$ or $d \div 2$.

1. Translate the following phrases into variable expressions.

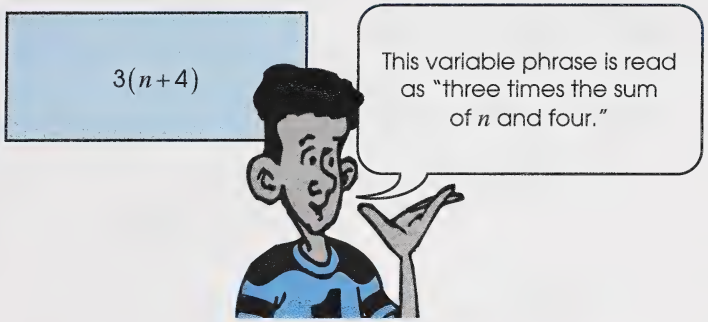
a. Marco's age increased by two years
b. twice the distance from Muriel's house to school
c. a number squared
d. five times the price of a car
e. a number decreased by two
f. a boy's age twelve years from now
g. Katya's earnings divided by two

Check your answers by turning to the Appendix.

You are now ready to translate variable expressions with more than one operation. Examples 3 and 4 show that the wording and the punctuation (or pauses) indicate the order of operations.

Example 3

Example 4



2. Write each of the following variable expressions in words.

a.	$2n-1$	**b.**	$2(n-1)$
c.	$4+5n$	**d.**	$(4+5)n$
e.	$6-3n$	**f.**	$(6-3)n$

3. Translate each of the following phrases into variable expressions. Let n be the number.

a. three more than the product of two and a number
b. four times a number, decreased by five
c. two times the sum of a number and five
d. four, times a number diminished by one

Check your answers by turning to the Appendix.

Evaluating Variable Expressions

In Activity 1 you evaluated mathematical expressions. In this activity you will evaluate variable expressions. You will discover that the value of a variable expression depends upon the value of the variable.

Example 1

The photo shows Joshanna and Tasha on holiday. Joshanna is three years older than her sister Tasha. How old will Joshanna be when Tasha is 10, 11, or 12 years old?

Solution

Step 1: Write a variable for Tasha's age.

Let Tasha's age be s.

Step 2: Write a variable expression for Joshanna's age.

Joshanna's age is $s+3$.

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Step 3: Find Joshanna's age when Tasha is 10, 11, or 12.

- If $s = 10$, then

$$\begin{aligned} & s + 3 \\ &= 10 + 3 \\ &= 13 \end{aligned}$$

When Tasha is 10, Joshanna will be 13.

- If $s = 11$, then

$$\begin{aligned} & s + 3 \\ &= 11 + 3 \\ &= 14 \end{aligned}$$

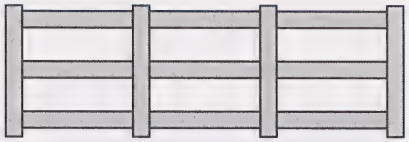
When Tasha is 11, Joshanna will be 14.

- If $s = 12$, then

$$\begin{aligned} & s + 3 \\ &= 12 + 3 \\ &= 15 \end{aligned}$$

When Tasha is 12, Joshanna will be 15.

Example 2



The number of crosspieces in a fence is three less than three times the number of posts. How many crosspieces are in the fence if there are 2, 3, or 4 posts?

Solution

Step 1: Write a variable for the number of posts.

Let the number of posts be n.

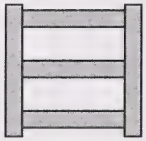
Step 2: Write a variable expression for the number of crosspieces.

The number of crosspieces is $3n - 3$.

Step 3: Find the number of crosspieces if there are 2, 3, or 4 posts.
Hint: Remember the order of operations.

- If $n = 2$, then

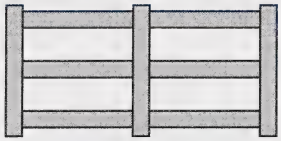
$$\begin{aligned} & 3n - 3 \\ &= 3 \times 2 - 3 \\ &= 6 - 3 \\ &= 3 \end{aligned}$$



If there are two posts, there are 3 crosspieces.

- If $n = 3$, then

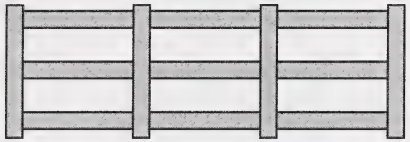
$$\begin{aligned} & 3n - 3 \\ &= 3 \times 3 - 3 \\ &= 9 - 3 \\ &= 6 \end{aligned}$$



If there are three posts, there are six crosspieces.

- If $n = 4$, then

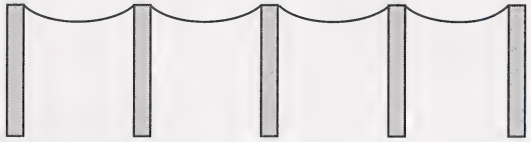
$$\begin{aligned} & 3n - 3 \\ &= 3 \times 4 - 3 \\ &= 12 - 3 \\ &= 9 \end{aligned}$$



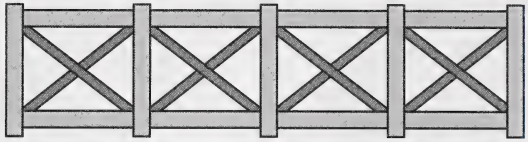
If there are four posts, there are nine crosspieces.

4. Linda's piggy bank contains only nickels. What is the value in cents of the coins in the bank if there are 10, 12, or 14 nickels?

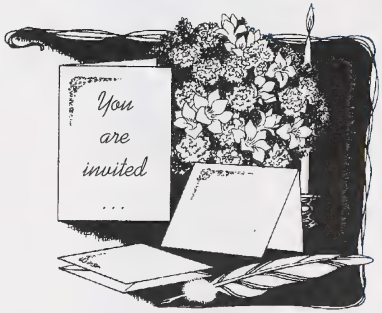
5. The number of chains in a fence is one less than the number of posts. How many chains are there in a fence if there are 2, 3, or 4 posts?



6. The number of crosspieces in a fence is four less than four times the number of posts. How many crosspieces are there in a fence if there are 2, 3, or 4 posts?



7. The cost in dollars for printing invitations is given by the expression $12 + 0.05n$, where n is the number of copies. What is the cost of printing 100, 200, or 300 invitations?



Check your answers by turning to the Appendix.

Sometimes it is useful to find and apply a pattern when you are evaluating a variable expression for several **consecutive** values of the variable.

Consecutive numbers are numbers written in order, such as 1, 2, 3.

Example 3

Evaluate the variable expression $a+5$, if $a=1,\ 2,\ 3,\ 4,\ 5,$ or 6.

Solution

Step 1: Evaluate the variable expression for the first three values.

If $a=1$, then	If $a=2$, then	If $a=3$, then
$a+5$	$a+5$	$a+5$
$=1+5$	$=2+5$	$=3+5$
$=6$	$=7$	$=8$

Step 2: Organize the values in a table to discover a pattern.

a	$a+5$	Pattern
1	6	
2	7	+1
3	8	+1

Step 3: Apply the pattern to find the last three values.

a	$a+5$	Pattern
1	6	
2	7	+1
3	8	+1
4	9	+1
5	10	+1
6	11	+1

I can see that using a pattern will be helpful when I am using consecutive numbers to evaluate an expression.

8. Complete the following tables. Evaluate the first three values and then use a pattern to help you find the last three values.

a.

a	$a+2$
1	
2	
3	
4	
5	
6	

b.

c	$2c$
1	
2	
3	
4	
5	
6	

c.

d	$3d+2$
1	
2	
3	
4	
5	
6	

d.

b	$4b-1$
1	
2	
3	
4	
5	
6	

9. How are the patterns in question 8 similar to the variable expressions? **Hint:** $a=1a$.

10. Each of the following algebraic expressions has a missing number shown by ☐. Use the table of values to find each value of ☐.

a.

k	☐ $k-3$
1	1
2	5
3	9
4	13

b.

y	☐ $y+1$
1	7
2	13
3	19
4	25

c.

d	☐ $d-1$
1	1
2	3
3	5
4	7

d.

x	☐ $x+4$
1	7
2	10
3	13
4	16

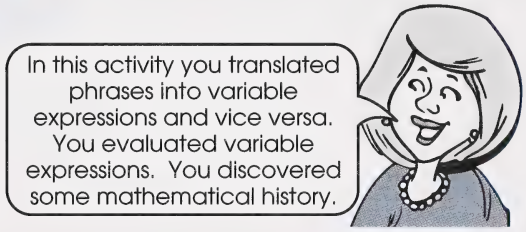
Check your answers by turning to the Appendix.

Did You Know?

Around the eighth century the Arabian Empire stretched from Spain to India, and Baghdad was a cultural and scientific centre. Arabian scholars studied mathematics and developed algebra.

The English word *algebra* was borrowed from the Arabic word *al-jabr* which was used in the title of a book written by the Arabian mathematician, Muhammad ibn Musa al-Khwarizmi (780–850).

Use the Internet to research another famous Arabian mathematician, Omar Khayaam (1048–1122).

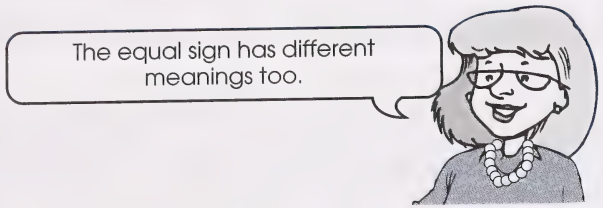

In this activity you translated phrases into variable expressions and vice versa. You evaluated variable expressions. You discovered some mathematical history.

Activity 3: Equations

What do the following expressions mean?

- I have the flu bug.
- I think my phone is bugged.
- I have the travel bug.
- Don't bug me.

The word *bug* can have many different meanings.



- Sometimes the equal sign is used as a symbol to signal that you are to find the value of an expression. The [=] key on a calculator is used this way. After you enter a mathematical expression, you press the [=] key to find the answer.

Example 1

[7] [+] [2] [=]

9.

- Sometimes the equal sign is used as a symbol to signal different forms of the same number.

Example 2

$2^3 = 8$

2^3 is the exponential form; 8 is the standard form.

- Sometimes the equal sign signals that two expressions have the same value.

Example 3

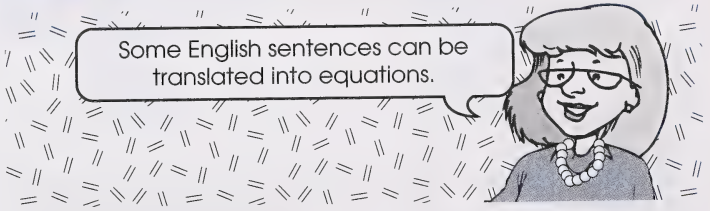
$7 + 2 = 12 - 3$

The mathematical expression "$7 + 2$" equals 9.
The mathematical expression "$12 - 3$" equals 9.

1. What does the equal sign signal in each of the following?

a. $4 + 3 + 2 = 3 \times 3$
b. $5 \times 3 + 2 = 17$
c. $\frac{5}{3} = 1\frac{2}{3}$

Check your answers by turning to the Appendix.



Any mathematical statement that contains an equal sign is called an **equation**.

Sentence	Equation
Three plus five is eight.	$3+5=8$
Eight minus two is the same as two times three.	$8-2=2\times3$
Ten divided by two gives five.	$10\div2=5$
Five more than the product of two and three results in eleven.	$2\times3+5=11$

Notice that *gives, results in, is the same as,* and *is* can all be translated by the equal sign, =.

2. Write each of the following sentences as an equation.

a. One plus three is four.

b. Three subtracted from eight gives five.

c. Twice the sum of one and three results in eight.

d. Three more than the product of two and four is the same as four less than the product of three and five.

Check your answers by turning to the Appendix.

Algebraic Equations

Some English sentences can be translated into **algebraic equations**.

Equations that contain variables are called algebraic equations.

Example

Write the following sentence as an algebraic equation:

The length decreased by 4 cm results in 25 cm.

Solution

Step 1: Determine the variable.

Let the length be ℓ.

Step 2: Write the algebraic equation.

$\ell-4=25$

3. Write each of the following sentences as an algebraic equation.

a. Twelve diminished by a number is four.

b. Six times a number is equal to forty-eight.

c. A number decreased by two results in seven.

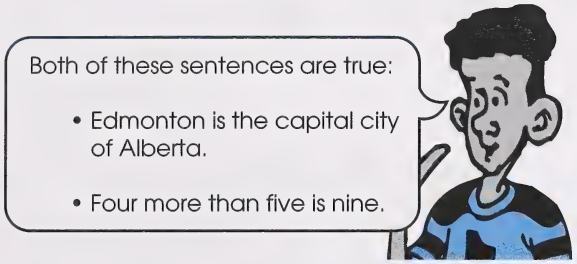
d. Bob's mass is 3 kg less than 52 kg.

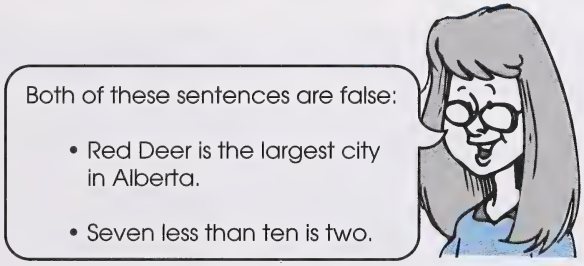
e. Marnie's money divided by two is $78.

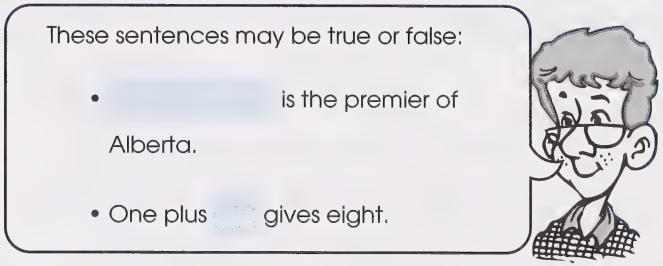
f. Nine less than twice the total number of newspapers delivered is twenty-seven newspapers.

Check your answers by turning to the Appendix.

Solving Equations by Inspection







4. Replace each to make each sentence true.

a. The capital of Canada is .

b. The boiling point of water is °C.

c. °C is the freezing point of water.

d. The is the smallest bird.

5. Replace each to make each equation true.

a. $\square + 5 = 13$

b. $5 + 4 = \square$

c. $8 - 3 = \square + 4$

d. $4 + 1 = \square - 2$

Check your answers by turning to the Appendix.

Each of the equations in question 5 is a **conditional equation**.

A conditional equation is only true for certain values of the variable.

The value of the variable that makes the equation true is called the **solution**.

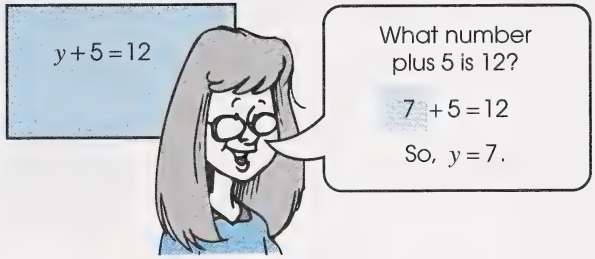
Finding the value of the variable that makes the equation true is called **solving the equation**.

You can solve many algebraic equations by simply looking at them. This is called the **inspection method**.

Example 1

Solve the equation $y+5=12$.

Solution



Note: You should check to decide if $y=7$ is the solution.

LS	RS
$y+5$	12
$=7+5$	
$=12$	
LS =	RS

LS means left side. RS means right side.

So, $y=7$.

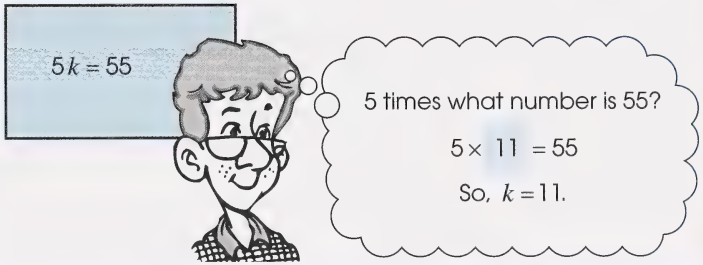
The check is called the **verification**. Checking the solution is called **verifying the solution**.

Example 2

Solve the equation $5k=55$. Then verify the solution.

Solution

Step 1: Solve the equation by using the inspection method.



Step 2: Verify the solution.

LS	RS
$5k$	55
$=5\times11$	
$=55$	

LS = RS

So, $k=11$.

6. Solve each of the following equations using the inspection method. Then verify each solution.

a. $n+8=12$ **b.** $p-5=5$ **c.** $4b=12$ **d.** $6+x=13$

Check your answers by turning to the Appendix.

Solving Equations by Guessing, Checking, and Revising

Some equations are more complicated and it is difficult to solve them using inspection. You may find that you need to use the guess, check, and revise method.

Example

Solve the equation $2(x+3)=8$. Then verify the solution.

Solution

Step 1: Make an initial guess, and check the guess.

Is $x=2$?

LS	RS
$2(x+3)$	8
$=2(2+3)$	
$=2\times5$	
$=10$	

LS ≠ RS

The initial guess is incorrect. The value is too high.

Step 2: Revise the guess and check again.

Is $x=1$?

LS	RS
$2(x+3)$	8
$=2(1+3)$	
$=2\times4$	
$=8$	

LS = RS

So, $x=1$.

7. Solve each of the following equations using the guess, check, and revise method. Then verify each solution.

a. $3a+2=14$ **b.** $2(a-7)=6$

c. $16=3s+1$ **d.** $3(a+4)=27$

Check your answers by turning to the Appendix.

Did You Know?

Hypatia (370–415) of Alexandria, Greece, was a mathematical scholar whose most significant work was in algebra.

Hypatia was also a teacher of high repute. She would summarize her lecture notes and give them to her students. These became the first mathematics textbook ever written!

Now Try This

Use a problem-solving strategy to answer the following question.

8. This is one of the problems that Hypatia asked her students to solve:

> Find a number that is the sum of two squares and whose square is also the sum of two squares.

Hypatia's problem has many solutions. See how many numbers you can find that solve this problem.

Check your answers by turning to the Appendix.

In this activity you discovered the meaning of an equation. You translated English sentences into equations. You solved algebraic equations by inspections, and by the guess, check, and revise method. You also learned some mathematical history.

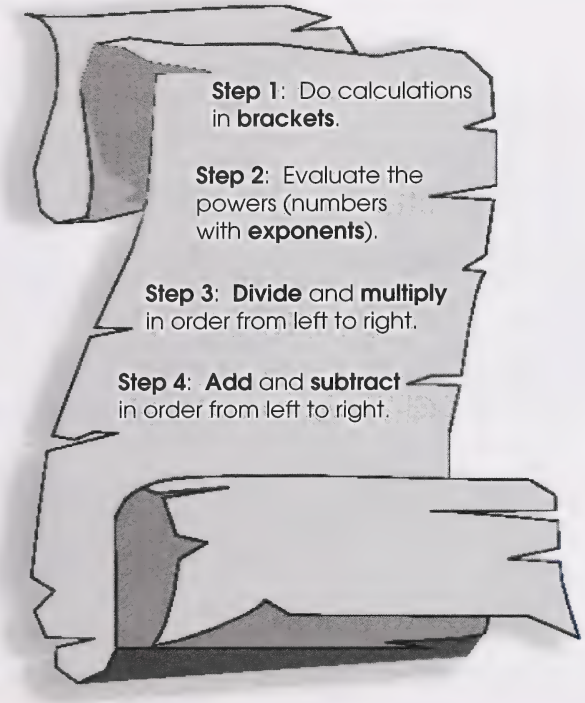
Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

Acronyms are sometimes used as memory aids. An acronym is a "word" formed from the initial letter or letters of words or phrases that you want to remember. The acronym **BEDMAS** will help you remember the rules for the order of operations.

Rules for the Order of Operations


Step 1: Do calculations in brackets.
Step 2: Evaluate the powers (numbers with exponents).
Step 3: Divide and multiply in order from left to right.
Step 4: Add and subtract in order from left to right.

The first letters of the following words were used to form the acronym BEDMAS:

B rackets

E xponents

D ivision / M ultiplication

A ddition / S ubtraction

Note: The words *multiply* and *divide* were switched in Step 3 in order to create the acronym. Remember, when there are both multiplication and division in an expression, you perform the operation that occurs first.

1. Use the paper-and-pencil method to evaluate each of the following mathematical expressions.

a.	$6+4\times2$	**b.**	$14+8\div2^2$	**c.**	$3\times(9+4)$
d.	$(6-5)\times3$	**e.**	$4\times5+7$	**f.**	$\left(7+3^2\right)\div2$
g.	$2+8\times4\div2$	**h.**	$10-8\div4\times3$	**i.**	$3^2+3\times4-1$

Check your answers by turning to the Appendix.

In this section you evaluated variable expressions and solved algebraic equations. If you are more comfortable using a [redacted] rather than a letter as a variable, feel free to rewrite the equation using this symbol.

If you use the geometric symbols, you may wish to photocopy the page of number tiles (tiles with the numbers 0 to 20) in the Appendix and cut out the tiles. When you rewrite the variable expression or equation, make each box big enough to place one of the tiles on it.

Example 1

Find the value of the variable expression $2n-4$ if $n=3$.

Solution

Step 1: Rewrite the variable expression using a .

$2\times$ -4

Step 2: Place the tile numbered 3 on the box and evaluate the expression.

$2\times 3 -4$

$=6-4$

$=2$

Follow the rules for the order of operations.

So, if $n=3$, then $2n-4=2$.

Example 2

Solve the equation $3y-1=8$. Then verify the solution.

Solution

Step 1: Rewrite the variable expression using a .

$3\times$ $-1=8$

Step 2: Use the guess, check, and revise method. Guess 2, place the tile with this value on the box, and check.

$3\times 2 -1=8$

LS	RS
$3\times 2-1$	8
$=6-1$	
$=5$	

LS $\neq$ RS

The initial guess is incorrect. The value is too low.

Step 3: Revise the initial guess. Guess 3, place the tile with the revised value on the box, and check.

$3\times 3 -1=8$

LS	RS
$3\times 3-1$	8
$=9-1$	
$=8$	

LS = RS

So, $y=3$.

2. Evaluate each of the following expressions.

   a. $5m$ if $m = 1, 2,$ or 3 **b.** $s + 2$ if $s = 3, 4,$ or 5
   c. $2x - 3$ if $x = 2, 4,$ or 6 **d.** $3m + 1$ if $m = 1, 2,$ or 3

3. Solve each of the following equations by the guess, check, and revise method. Then verify each solution.

   a. $12 + a = 15$ **b.** $2n = 18$ **c.** $b - 5 = 3$
   d. $3a + 2 = 14$ **e.** $2x - 1 = 9$ **f.** $2(a + 5) = 14$

Check your answers by turning to the Appendix.

Enrichment

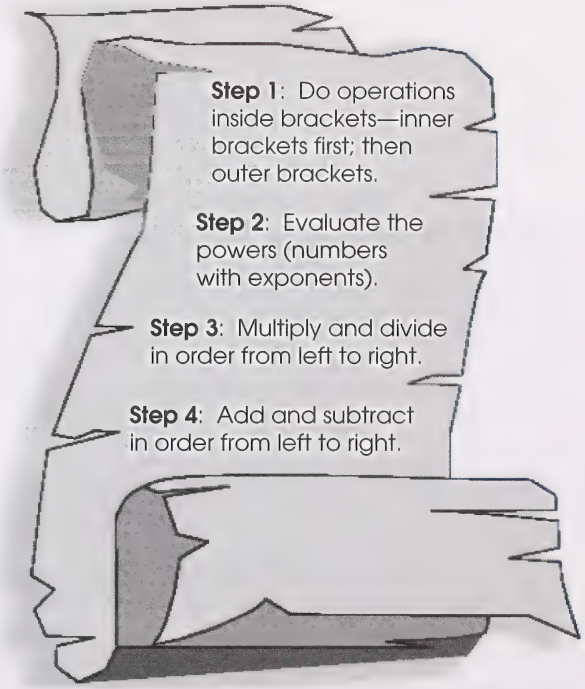
In this section you used the rules for the order of operations to evaluate mathematical expressions. Some of the mathematical expressions you evaluated included brackets.

There are three different kinds of brackets that can be used in mathematical expressions:

- These brackets, ( ), are called parentheses.
- These brackets, [ ], are called square brackets.
- These brackets, { }, are called braces.

When there is a set of brackets inside another set of brackets, the following rules for operations are used.

Rules for the Order of Operations


Step 1: Do operations inside brackets—inner brackets first; then outer brackets.
Step 2: Evaluate the powers (numbers with exponents).
Step 3: Multiply and divide in order from left to right.
Step 4: Add and subtract in order from left to right.

Example 1

Evaluate the mathematical expression $4 \times [4 \times (4 + 4)]$.

Solution

$4\times[4\times(4+4)]$ ← Perform operations in the inner brackets.

$= 4\times[4\times 8]$ ← Perform operations in the outer brackets.

$= 4\times 32$ ← Multiply.

$= 128$

Example 2

Evaluate $10\div[8-(6-3)]$.

Solution

$10\div[8-(6-3)]$ ← Perform operations in the inner brackets.

$= 10\div[8-3]$ ← Perform operations in the outer brackets.

$= 10\div 5$ ← Divide.

$= 2$

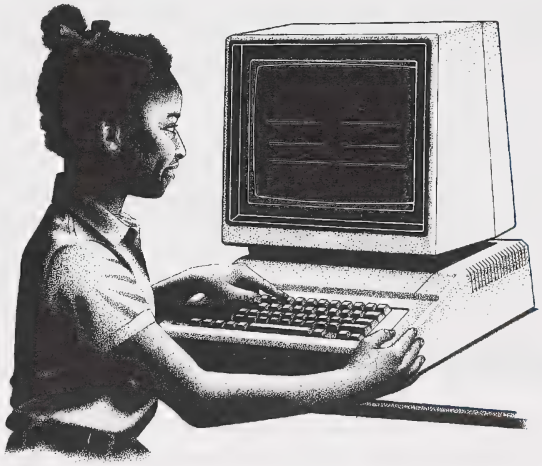
1. Find the value of each of the following mathematical expressions.

a. $10-[(4+8)\div 3]$ **b.** $10\times[6\div(8-5)]$

c. $[(3+2)\times(5-3)]\div 2$ **d.** $4+6\times[2\times(6-3)]$

Check your answers by turning to the Appendix.

In this section you evaluated variable expressions. You can use a computer and a spreadsheet program such as *ClarisWorks*™ to evaluate a variable expression for several values of the variable.



Example 3

Use a spreadsheet program such as *ClarisWorks*™ to evaluate the expression $4n-2$ if $n=1.8,\ 2.7,\ 5.1,$ or 8.3.

Solution

Step 1: Label the cells in the first column on the spreadsheet.
Hint: The cell address at the top of the spreadsheet shows which cell is active. There is also a border around the active cell. You don't type directly in this cell; instead you use the entry bar at the top of the spreadsheet.

A4	× √	4n – 2		
	A	B	C	D
1	Evaluate 4n – 2			
2				
3	n			
4	4n – 2			
5				

Step 2: Begin with the B3 cell and enter the first value of the variable, 1.8. Next, in the B4 cell, enter the calculations you wish the computer to do:

This means the resulting number in the B4 cell is 2 less than 4 times the number in the B3 cell.

The computer will do the calculations and the number **5.2** will appear in the B4 cell.

B4	× √	= 4* B3 – 2		
	A	B	C	D
1	Evaluate 4n – 2			
2				
3	n	1.8		
4	4n – 2	5.2		
5				

So, $4n-2=5.2$ if $n=1.8$.

Step 3: Delete the number in the B3 cell and enter the second value, 2.7. The computer will do the calculations and the number **8.8** will appear in the B4 cell.

B4	× √	= 4* B3 – 2		
	A	B	C	D
1	Evaluate 4n – 2			
2				
3	n	2.7		
4	4n – 2	8.8		
5				

So, $4n-2=8.8$ if $n=2.7$.

Step 4: Delete the number in the B3 cell and enter the third value, 5.1. The computer will do the calculations and the number **18.4** will appear in the B4 cell.

B4	× √	= 4* B3 – 2		
	A	B	C	D
1	Evaluate 4n – 2			
2				
3	n	5.1		
4	4n – 2	18.4		
5				

So, $4n-2=18.4$ if $n=5.1$.

Use a computer and a spreadsheet program such as *ClarisWorks*™ to answer question 2.

2. Evaluate each of the following expressions.

a. $5x+2$, if $x = 2.5,\ 3.8,$ or 5.7
b. $7s-1$, if $s = 1.9,\ 2.4,$ or 4.2
c. $8.2n+3.2$, if $n = 10,\ 12,$ or 20

Check your answers by turning to the Appendix.

Did You Know?

Sir Isaac Newton[1]

Isaac Newton was born on a farm in England on Christmas Day, 1642. His father died before he was born, and Isaac was brought up by his grandmother. He was never very strong but was able to go to a country school.

Young Isaac liked to experiment and to make things. Once he made a little toy mill that could grind wheat into flour. Real mills of his day were sometimes driven by mules or oxen, but Isaac's was driven by a mouse! He also made a wooden clock that worked by water.

Isaac's uncle saw that the boy was very bright and thought he should go to college. Fortunately, his family was able to send him. There, at the age of eighteen, he studied chemistry at first, but he soon began to get interested in mathematics. Isaac read Euclid's *Elements*, which he found to be much too easy for him. Then he read the book *Geometry*, by Descartes, a mathematician whose work had been done about the time Isaac was born. This he found more difficult. Next he read works by a few other great mathematicians.

Isaac Newton was so bright that after these readings, he was able to start doing all sorts of hard mathematical problems by himself. When he was twenty-three years old, his college had to close because of the plague. While Newton was at home that year, he worked on calculus, the subject for which he is famous. He also began his great study of the way Earth and the Moon move. He decided that they move as they do because of what he called "gravity."

When Newton had a problem to think about, he put everything else out of his mind. Once he was thinking about a problem and walking his horse up a hill at the same time. The horse somehow slipped out of the bridle and ran away. Newton didn't know the horse was gone until, at the top of the hill, he tried to jump on the horse!

Although Newton was one of the greatest mathematicians of all time, he was a very modest man. He felt he owed much to those mathematicians who lived before him. This is what he meant when he said, "If I have seen farther than others, it is because I have stood on the shoulders of giants."

Newton's mathematical ideas are among the most difficult ever thought about and also among the most useful. Modern science owes much to Newton. Many of the things you enjoy today were made possible by Newton's mathematics.

[1] Reprinted with permission from *Mathematical History: Activities, Puzzles, Stories, and Games* by Merle Mitchell. Copyright 1978 by the National Council of Teachers of Mathematics.

Conclusion

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In this section you discovered how algebra is related to arithmetic. You translated English phrases and sentences into mathematical expressions, variable expressions, and equations. You evaluated mathematical and variable expressions, and you used informal methods to solve and verify equations.

You discovered that mathematical symbols are used in the language of mathematics.

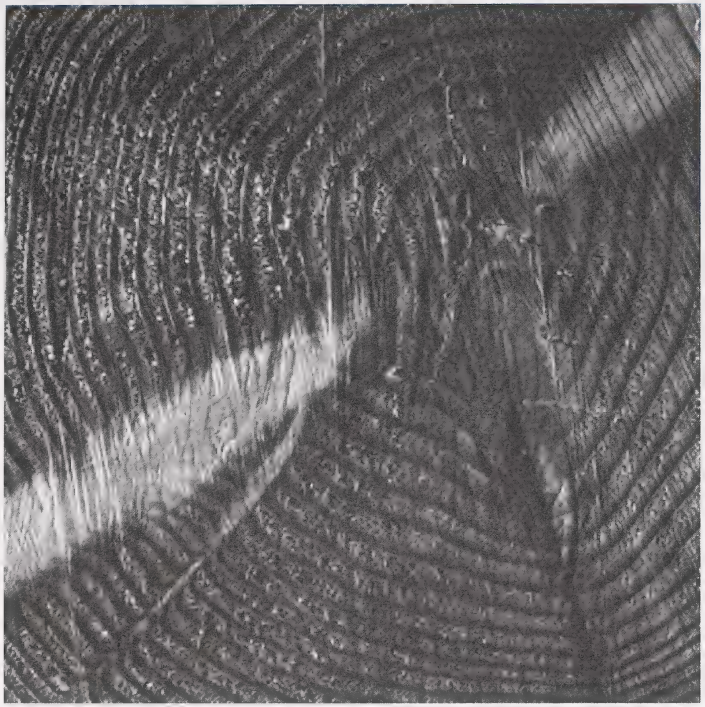
Symbols are also used in the everyday world to represent ideas or to make communication easier. Look at the labels of several products around your home. What types of symbols are on some packages? What do these symbols mean?

Assignment

Assignment Booklet

You are now ready to complete the module assignment for Section 1.

Section 2: Patterns and Relations



Have you ever looked at the cross section of a tree? There is an interesting pattern of rings. Scientists have discovered that the age of a tree is related to the number of rings. Other patterns indicate periods of slow or rapid growth, disease, or fire damage.

Mathematics involves the study of patterns. The ability to find and apply patterns is an important mathematical skill.

In this section you will describe patterns and relations. You will also interpret graphs of relations.

Activity 1: Describing Patterns and Relations

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Have you been to Writing-on-Stone Provincial Park in southern Alberta? This photo shows some of the "writings." People have tried to describe the world for thousands of years.

Non-numerical Relations

In the first part of this activity you will describe non-numerical patterns and relations. You will discover that there are a variety of ways to show how two sets of numbers are related.

Example

When people travel to another country, they need to know what kind of money or currency is used there.

The following table matches currencies with countries.

Country	Currency
Germany	marks
United States	U.S. dollars
Australia	Australian dollars
Britain	pounds
Italy	lire
Japan	yen
France	francs

Another way to describe the relationship between the countries and the currencies is to list the set of **ordered pairs**.

$\{$(Germany, marks), (United States, U.S. dollars), (Australia, Australian dollars), (Britain, pounds), (Italy, lire), (Japan, yen), (France, francs)$\}$

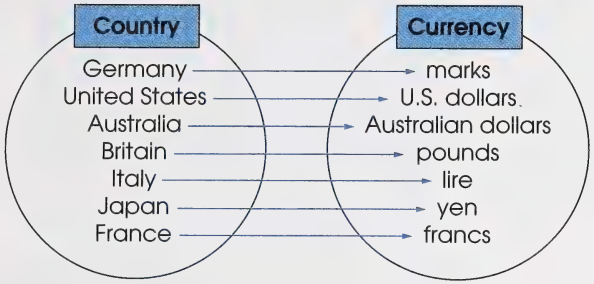
The outside brackets indicate a set.

An ordered pair is a pair for which the order is important.

In the given example, the first member of each ordered pair is the name of a country and the second member of each ordered pair is the name of a currency: (country, currency).

A set of ordered pairs is called a **relation**.

Another way to show the relationship between the countries and the currencies is to draw and label an arrow diagram.



This arrow diagram shows a **mapping**.

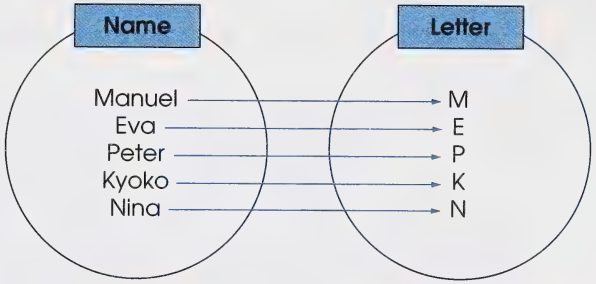
A mapping is a method of showing a relationship between pairs.

In the arrow diagram each country is paired with or **mapped onto** a currency.

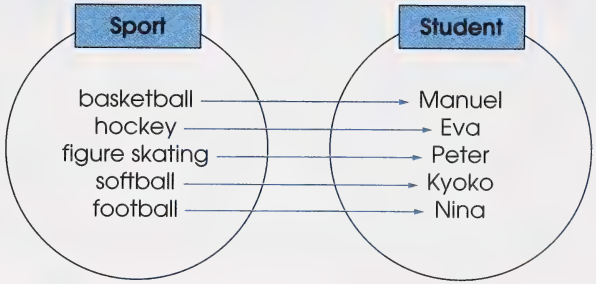
The given arrow diagram shows the relation rule "calls its currency." For example, one of the pairings is "Germany *calls its currency* marks."

1. List the set of ordered pairs for each of the following arrow diagrams.

a.



b.



2. Make up a relation rule for each of the arrow diagrams in question 1.

3. Draw an arrow diagram for each set of ordered pairs.

   a. {(British Columbia, Victoria), (Alberta, Edmonton), (Saskatchewan, Regina), (Manitoba, Winnipeg)}

   b. {(mass, gram), (capacity, litre), (length, metre)}

4. List the ordered pairs for each of the tables.

   a.

Student's Name	Mother's Name
Jacques	Marie
Franz	Joanne
Jeanne	Yvonne
Mary	Ruth
Ichiro	Isabelle

   b.

Unit	Symbol
centimetres	cm
degrees Celsius	°C
kilograms	kg
millilitres	mL

Check your answers by turning to the Appendix.

Numerical Relations

You have worked with relations between people, places, and things. Now you will explore relations involving only numbers.

Example 1

The value in cents of several dimes is ten times the number of dimes. If there is one dime, the value is 10¢. If there are two dimes, the value is 20¢. If there are three dimes, the value is 30¢. If there are four dimes, the value is 40¢. Describe the relationship between the number of dimes and the value of the dimes.

Solution

Method 1: Making a Table

One way to match the number of dimes and their value is to make a table.

Number of Dimes	Value of the Dimes (¢)
1	10
2	20
3	30
4	40
...	...

← The three dots indicate the pattern continues.

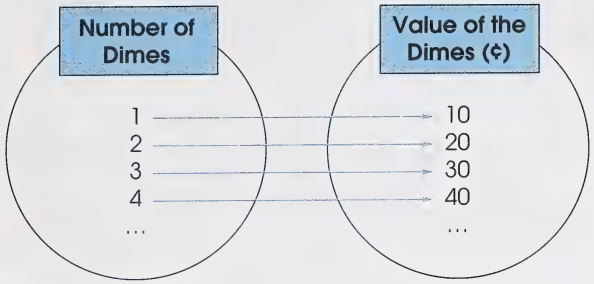
Method 2: Writing a Set of Ordered Pairs

Another way to describe the relationship is to write the set of ordered pairs.

$\{(1,10),\ (2,20),\ (3,30),\ (4,40),\ldots\}$ ← The three dots indicate the pattern continues.

Method 3: Using an Arrow Diagram

Another way to match the number of dimes with their value is to draw and label an arrow diagram.



Method 4: Writing a Rule

Another way to describe the relationship is to write a relation rule. To find the relation rule, look for a pattern.

Number of Dimes	Value of the Dimes (¢)	Pattern
1	10	10×1
2	20	10×2
3	30	10×3
4	40	10×4
...	...	
x	$10x$	$10\times x=10x$

The relation rule may also be written as $x \longrightarrow 10x$.

Example 2

Draw a diagram for the relation rule $x \longrightarrow x+5$, where $x=1,\ 2,\ 3$.

Solution

If $x=1$, then

$x+5$
$=1+5$
$=6$

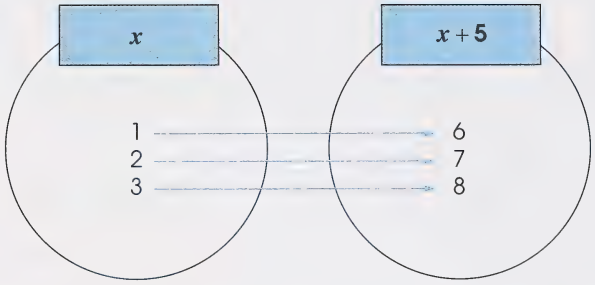
If $x=2$, then

$x+5$
$=2+5$
$=7$

If $x=3$, then

$x+5$
$=3+5$
$=8$

This is the arrow diagram.



5. Draw an arrow diagram for each of the following relation rules.

a. $x \longrightarrow x-1$, where $x=1,\ 2,\ 3,\ 4$
b. $x \longrightarrow 3x$, where $x=1,\ 3,\ 5,\ 7$

Check your answers by turning to the Appendix.

6. a. Make a chart like the following. Complete the chart by writing the number of squares and the number of dots for each shape.

Shape	Number of Squares	Number of Dots

b. Find the pattern. If the pattern is continued, how many dots will there be in a shape which has five squares? six squares? seven squares?

c. Check your predictions in question 6.b. by drawing the shapes.

d. If the pattern is continued, how many dots will there be in the shape which has n squares?

e. Write a relation rule to describe the pattern.

f. Use the relation rule to find how many dots there will be in the shape which has 25 squares.

7. a. Make a chart like the following. Complete the chart by writing the total number of toothpicks and the number of vertical toothpicks for each shape.

Shape	Total Number of Toothpicks	Number of Vertical Toothpicks

b. Find the pattern. If the pattern is continued, what will be the number of vertical toothpicks if there is a total of 12 toothpicks? 15 toothpicks? 18 toothpicks?

c. Check your prediction in question 7.b. by modelling the shapes with toothpicks or by making diagrams.

d. If the pattern is continued, how many vertical toothpicks will there be in the shape which has a total of x toothpicks?

e. Write a relation rule to describe the pattern.

f. Use the relation rule to find how many vertical toothpicks there will be in the shape which has 30 toothpicks.

8. a. Make a chart like the following. Complete the chart by writing the number of triangles and the total number of toothpicks.

Shape	Number of Triangles	Number of Toothpicks

b. Find the pattern. If the pattern is continued, how many toothpicks will be needed to make four triangles? five triangles? six triangles?

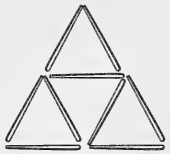
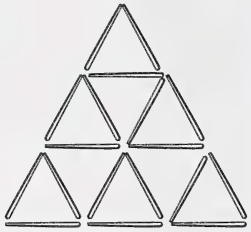
c. Check your prediction in question 8.b. by modelling the shapes with toothpicks or by drawing diagrams.

d. If the pattern is continued, how many toothpicks will be needed to make n triangles?

e. Write a relation rule to describe the pattern.

f. Use the relation rule to find how many toothpicks will be needed if there are 27 triangles.

9. a. Make a chart like the following. Complete the chart by writing the number of toothpicks on the base of the shape and the number of small triangles created. **Hint:** The base is the bottom of the triangle (the side the triangle rests on).

Shape	Number of Toothpicks on the Base	Number of Small Triangles
		
		

b. Find the pattern. If the pattern is continued, how many small triangles will there be if there are four toothpicks on the base? if there are five toothpicks on the base? if there are six toothpicks on the base?

c. Check your prediction in question 9.b. by modelling the shapes with toothpicks or by drawing diagrams.

d. If the pattern is continued, how many triangles can be created with x toothpicks on the base?

e. Write a relation rule to describe the pattern.

f. Use the relation rule to find how many triangles can be created with 14 toothpicks on the base.

10. Ruth works on a construction site.

The following table gives her pay for various numbers of hours of work.

Number of Hours	Pay ($)
1	15.25
2	30.50
3	45.75
4	61.00

a. How much will Ruth earn if she works for 5 h?

b. How much will Ruth earn if she works for x hours?

c. Write a relation rule to describe how the number of hours and the amount of pay are related.

d. Use the relation rule to find out how much Ruth earns in a week. (An average work week is 40 hours.)

11. Sylvia likes to go horseback riding.

The following table gives the cost for various riding times.

Riding Time (h)	Cost $
1	6
2	8
3	10
4	12
5	14

a. If Sylvia rides for 6 h, what will the cost be?

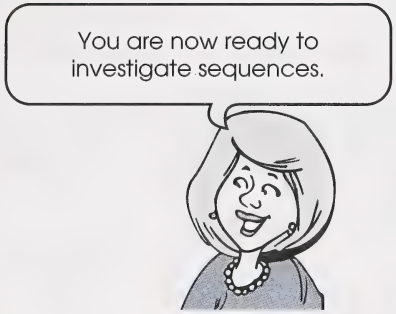
b. If Sylvia rides for n hours, what will the cost be?

c. Write a rule to describe how the number of hours of riding time and the cost are related.

d. If Sylvia rides for 1.5 h on Monday, 3 h on Tuesday, and 6 h on Thursday, what will the cost be ?

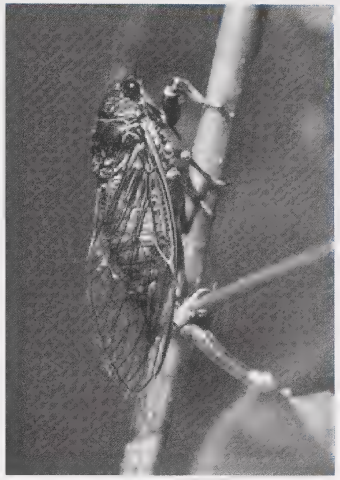
Check your answers by turning to the Appendix.

Sequences



In mathematics a **sequence** is a set of numbers that is ordered according to some rule.

Example 1



The cicada is an insect that spends most of its life underground, feeding on root sap. At regular intervals, cicadas emerge above ground to lay eggs and die. Then the cycle repeats itself. In one area cicadas last emerged in 1990. Before that they emerged in 1973, 1956, 1939, 1922,... .

When will the insects appear again in this area? Why is this insect sometimes called "the seventeen-year locust"?

Solution

Step 1: Study the sequence and look for a pattern.

..., 1922, 1939, 1956, 1973, 1990,...

+17 +17 +17 +17

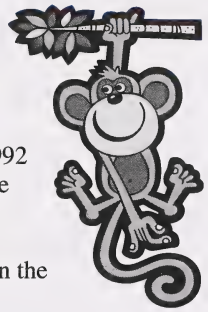
Step 2: Continue the pattern.

..., 1922, 1939, 1956, 1973, 1990, 2007,...

+17 +17 +17 +17 +17

The cicadas will next emerge in 2007. This insect is sometimes called "the seventeen-year locust" because it emerges every 17 years.

12. In Chinese culture, years are named after different animals. These years occur at regular intervals. The year 1992 was the year of the Monkey.



a. If 1932, 1944, 1956, 1968, 1980, and 1992 were years of the Monkey, when will the next three years of the Monkey occur?

b. What was the first year of the Monkey in the twentieth century?

Check your answers by turning to the Appendix.

In working with sequences, it is often useful to refer to **terms**.

The first number in a sequence is called the first term, the second number is called the second term, and so on.

You can use relations to help you predict a term in a sequence.

Example 2

1st term	2nd term	3rd term	4th term	5th term		50th term
↓	↓	↓	↓	↓		↓
2,	4,	6,	8,	10,...,		

What is the 50th term in this sequence?

Solution

Step 1: Make a table and find a relation rule to describe the pattern.

Position of Term	Term	Pattern
1st	2	← 2×1
2nd	4	← 2×2
3rd	6	← 2×3
4th	8	← 2×4
5th	10	← 2×5
...	...	
nth	$2n$	← $2\times n=2n$

The relation rule is $n \longrightarrow 2n$.

Step 2: Use the relation rule to find the 50th term.

If $n = 50$, then

$$\begin{aligned} &2n \\ &= 2\times 50 \\ &= 100 \end{aligned}$$

So, the 50th term is 100.

13. Find the 6th, 7th, 8th, and nth terms in each of the following sequences.

a. 3, 6, 9, 12, 15, ___, ___, ___, ...,

b. 1, 3, 5, 7, 9, ___, ___, ___,...,

c. 4, 7, 10, 13, 16, ___, ___, ___,...,

d. 1, 4, 9, 16, 25, ___, ___, ___,...,

e. 2, 5, 10, 17, 26, ___, ___, ___,...,

Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following question.

14. The amount of water that water lilies cover on a lake doubles every day. If it takes 60 days for the lake to become covered with water lilies, on what day is 0.25 of the lake covered?

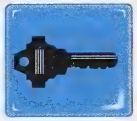
Check your answer by turning to the Appendix.

Did You Know?

The French mathematician Marin Mersenne (1588–1648) thought that powers of 2 were intriguing. He examined prime numbers and found a pattern.

$$3 = 2^2 - 1$$
$$7 = 2^3 - 1$$
$$31 = 2^5 - 1$$
$$127 = 2^7 - 1$$
$$8191 = 2^{13} - 1$$

The numbers 3, 7, 31, 127, and 8191 are prime numbers. The exponents 2, 3, 5, 7, and 13 are also prime numbers.



The prime numbers 3, 7, 31, 127, 8191,..., $2^n - 1$, where n is a prime are **Mersenne primes**.

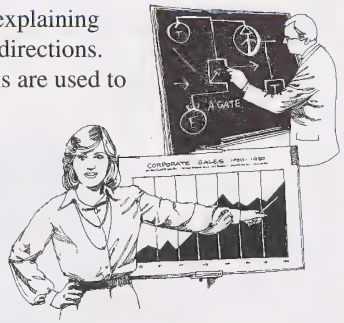
Mersenne thought there were only eleven Mersenne primes. It has since been shown that two of the numbers that he thought were Mersenne primes are actually composite numbers. Other Mersenne primes have also been found.

Use the Internet to discover how many Mersenne primes are known today and the exponent of the largest known Mersenne prime. **Hint:** At the time this course was published, there were several sites on prime numbers that provided this information. You may use one of the many on-line search engines to find these sites.

In this activity you discovered that numerical relations can be described by a table, a set of ordered pairs, an arrow diagram, or a rule. You explored sequences and predicted terms in the sequences. You continued to solve non-routine problems. You read about Mersenne primes, and you used the Internet to answer questions about Mersenne primes.

Activity 2: Graphs of Relations

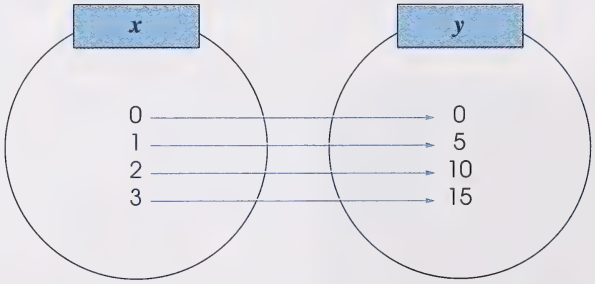
People often use visual aids when explaining something. Maps are used to give directions. Diagrams are used to teach. Graphs are used to show trends.



In the previous activity you used arrow diagrams to show the relationship between two sets of numbers. Arrow diagrams of a relation are effective because they give a visual representation.

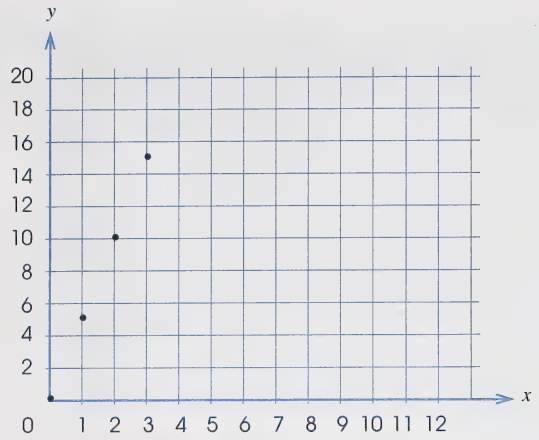
Graphs are also useful for describing relations. The graph in Example 2 shows the same relation as the arrow diagram in Example 1.

Example 1



In this arrow diagram, the first set of numbers is represented by the variable x, and the second set of numbers is represented by the variable y. The relation can be described by the ordered pairs $(0, 0)$, $(1, 5)$, $(2, 10)$, and $(3, 15)$.

Example 2



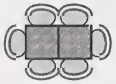
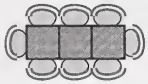
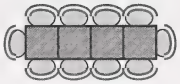
The dots on this graph indicate the ordered pairs $(0,0)$, $(1,5)$, $(2,10)$, and $(3,15)$.

You have been making graphs for several years so you probably know how to read graphs and plot graphs. However, if you need extra help with these skills or a quick review, you may wish to turn to the Follow-up Activities before continuing.

In this activity you will use graphs to examine patterns and make predictions.

Example 3

Manuel owns a restaurant. He sometimes puts tables together in order to seat different numbers of people. Some of the possible arrangements are shown in the following chart.

Arrangements	Number of Tables	Number of Chairs
	1	4
	2	6
	3	8
	4	10

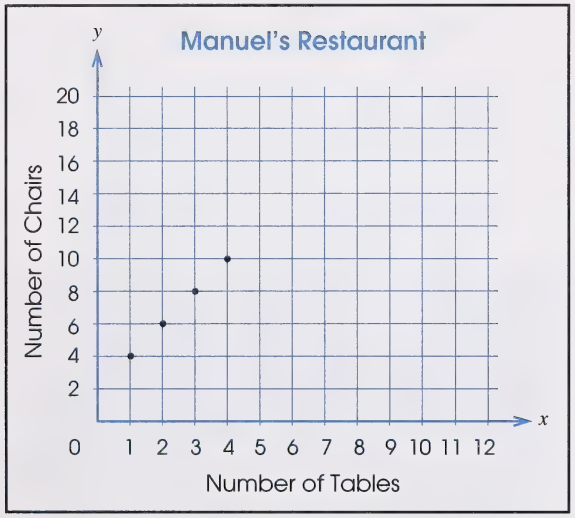
Use a graph to show the relationship between the number of tables and the number of chairs. Use the graph to predict the number of chairs with 5 tables, 6 tables, or 7 tables.

Solution

Step 1: Use the chart to write a set of ordered pairs.

$$\{(1,4),\ (2,6),\ (3,8),\ (4,10),\ldots\}$$

Step 2: Find an appropriate scale for each axis and plot the points. **Hint:** Do not connect the dots because it is not possible to have a fraction of a table or chair.

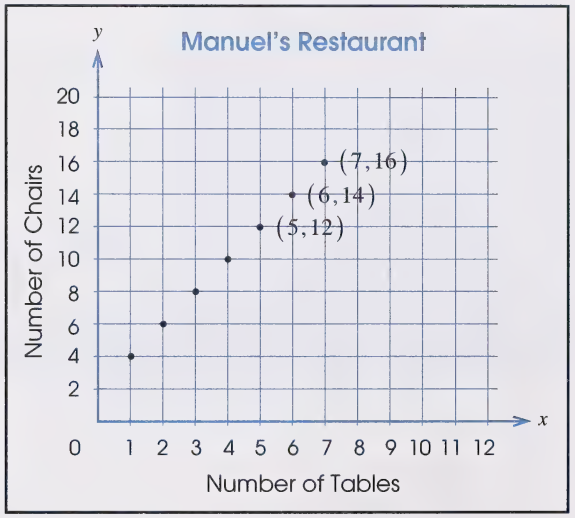


This is the graph of the relation.

Step 3: Use the graph to make the predictions. **Hint:** The dots lie in a straight line.

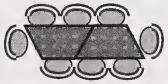
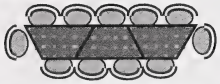
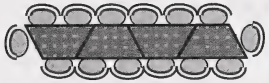
To predict the number of chairs with 5, 6, or 7 tables, continue the pattern of dots.

Each new dot is 1 unit to the right and 2 units up.



For 5 tables there will be 12 chairs. For 6 tables there will 14 chairs. For 7 tables there will be 16 chairs.

1. A school has special tables that can be put together to seat different numbers of students.

Arrangements	Number of Tables	Number of Chairs
	1	5
	2	8
	3	11
	4	14

a. Use a graph to show the relationship between the number of tables and the number of chairs.

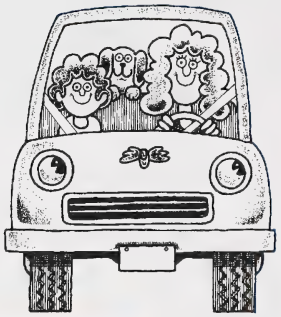
b. Predict the number of chairs for 5, 6, or 7 tables.

Check your answers by turning to the Appendix.

In the graph in Example 3, Manuel's Restaurant, the dots were not connected because only whole numbers of tables and chairs were used. When you draw a graph with units such as time, it is possible to have fractional units; therefore, the dots are connected.

Example 4

Rachel is going on a trip by car.



The following table shows the number of kilometres she travels after leaving home. The table pairs the distance travelled with the time taken.

Time Elapsed (min)	Distance Travelled (km)
10	14
20	28
30	42
40	56
50	70

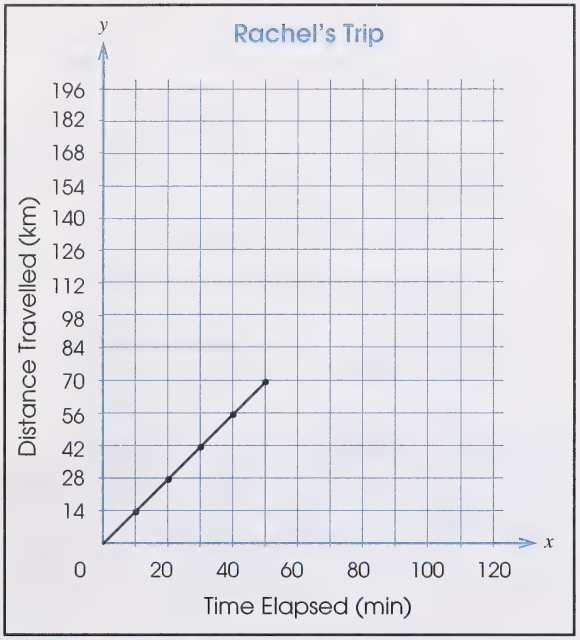
Draw a graph to represent the relation. Use the graph to predict how far Rachel will travel in 100 min if this pattern continues.

Solution

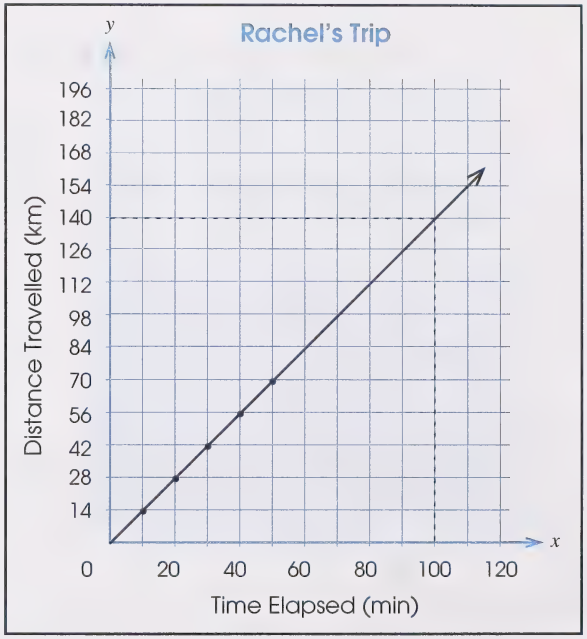
Step 1: Use the table to write the ordered pairs.

$$\{(10,14),\ (20,28),\ (30,42),\ (40,56),\ (50,70)\}$$

Step 2: Find an appropriate scale for each axis. Plot the points and connect the dots. **Hint:** You can connect the points because you can have fractional units of time.



The arrowhead on the line indicates the pattern continues.



Step 3: Use the graph to make the prediction. **Hint:** The points lie in a straight line. To predict how far Rachel will travel in 100 min, extend the line.

Rachel will travel 140 km in 100 min if this pattern continues.

The process of finding a value by following a pattern and going beyond the values already known is called **extrapolation**.

2. Lesley is a welder. She is paid an hourly wage. The following table shows her rate of pay.

Number of Hours Worked	Earnings ($)
1	17
2	34
3	51
4	68

a. Make a graph to describe the relation.
b. Predict the amount Lesley will earn when she works 5.5 h.

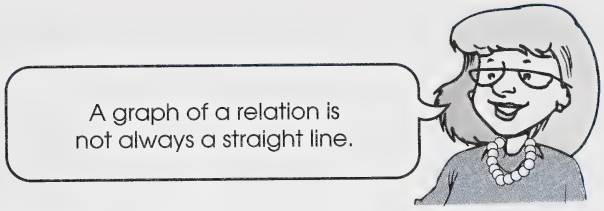
3. A biologist measured the teeth of many white sharks. He also measured the body lengths of the sharks. Here is some of the data the scientist found.

Length of Body (m)	Length of Longest Tooth (cm)
1.4	1.5
2.4	2.6
3.0	3.4
3.6	4.0

a. Graph the relation.

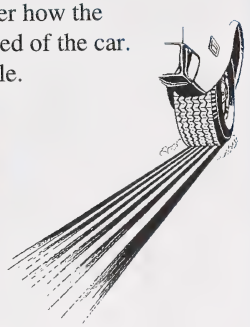
b. A 6.40-m white shark was caught by Karen and Joe. How long would you expect the shark's longest tooth to be?

Check your answers by turning to the Appendix.



Example 5

A police officer did an experiment to discover how the braking distance of a car is related to the speed of the car. The results are recorded in the following table.



Speed (km/h)	Braking Distance (m)
25	3
50	12
75	27
100	48

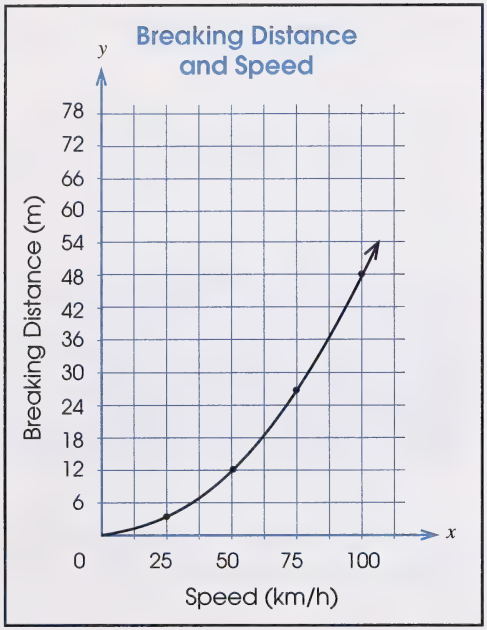
Draw a graph of the relation. Use the graph to calculate the speed if the braking distance is 36 m.

Solution

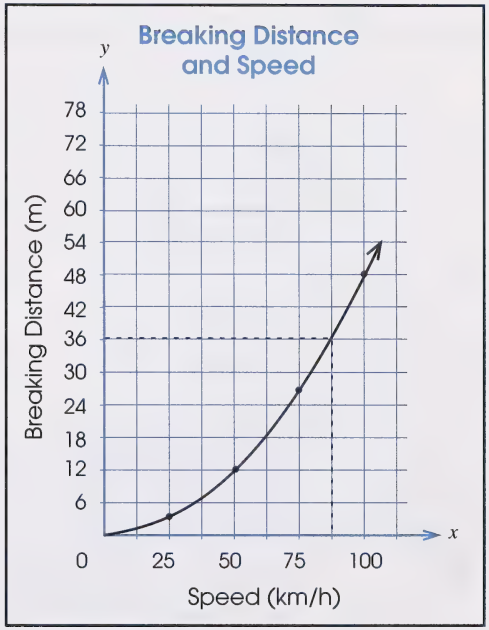
Step 1: Use the table to write the ordered pairs.

$$\{(25,3),\ (50,12),\ (75,27),\ (100,48)\}$$

Step 2: Find an appropriate scale for each axis. Plot the points and connect the dots with a smooth curve.



The arrowhead on the graph indicates the pattern continues.



The grid line between 75 and 100 on the x-axis is 87.5.

The speed was 87.5 km/h.

Step 3: Use the graph to find the speed if the braking distance is 36 m. **Hint:** The dotted lines show the x-value that matches a y-value of 36.

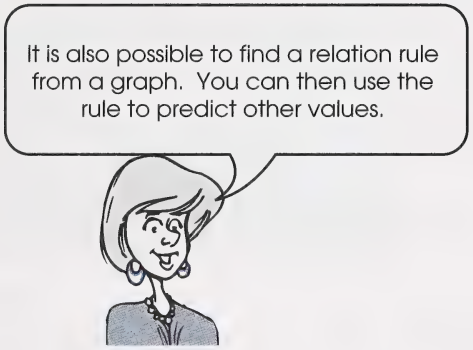
The process of finding a value between two other known values is called **interpolation**.

4. Helia is a runner. The following table shows her heart rate at different times after running.

Heart Rate (beats/min)	Time After Running (min)
180	0
150	2
120	4
100	6
90	8
80	10

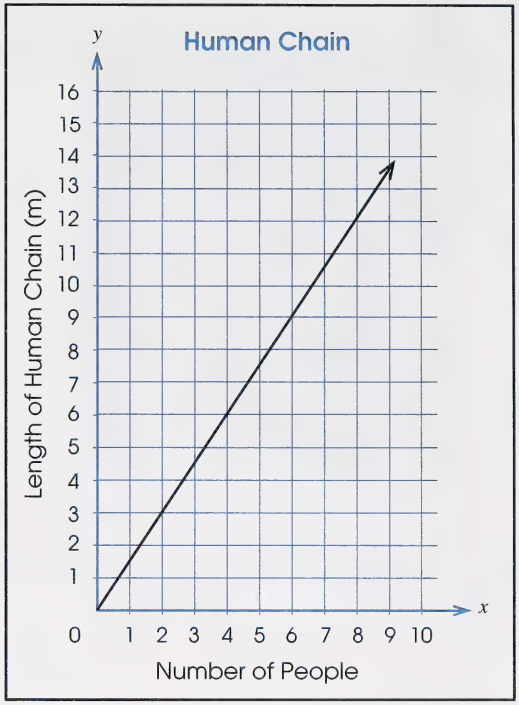
a. Make a graph to describe the relation.
b. Calculate Helia's heart rate 5 min after running.

Check your answers by turning to the Appendix.



Example 6

Percy and Natasha found the following graph in a magazine.



Based on the graph, how long a chain would the 30 students in their mathematics class be able to form?

Solution

Step 1: Use the graph to find the length of a human chain with 1, 2, 3, 4, or 5 people. A table is a good way to organize your work.

Number of People	Length of the Human Chain (m)
1	1.5
2	3
3	4.5
4	6
5	7.5

Step 2: Find a pattern and write the relation rule.

Number of People	Length of the Human Chain (m)	Pattern
1	1.5	← 1.5×1
2	3	← 1.5×2
3	4.5	← 1.5×3
4	6	← 1.5×4
5	7.5	← 1.5×5
...	...	
n	$1.5n$	← $1.5\times n = 1.5n$

So, the relation rule is $n \longrightarrow 1.5n$.

Step 3: Use the relation rule to find the length of chain the classroom of students could form.

If $n = 30$, then

$$\begin{aligned} & 1.5n \\ &= 1.5 \times 30 \\ &= 45 \end{aligned}$$

So, 30 students can make a chain 45 m long.

5. On January 1, Herbert has $250 saved. He makes a New Year's resolution to save a regular amount from his weekly allowances in order to buy a bicycle.

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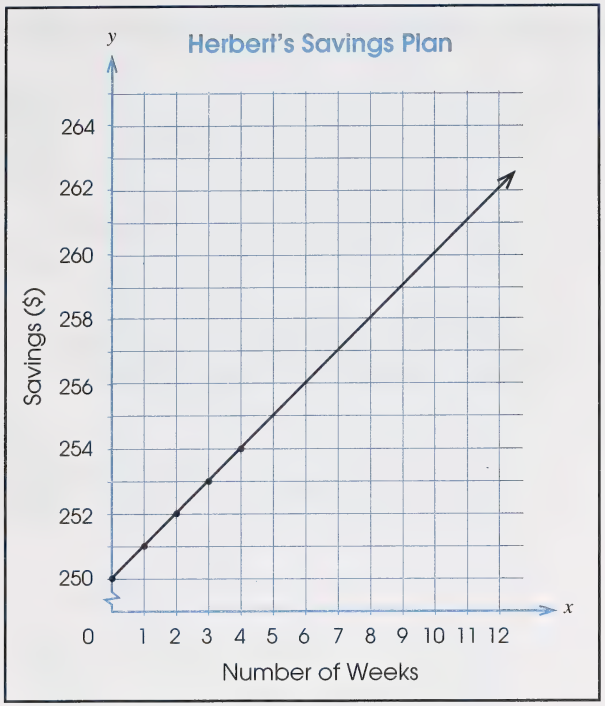
He makes the following graph to show how his savings will grow.



a. Use the graph to find a relation rule for Herbert's pattern of saving.

b. Use the relation rule to find how much Herbert could save in a year. **Hint:** There are 52 weeks in a year.

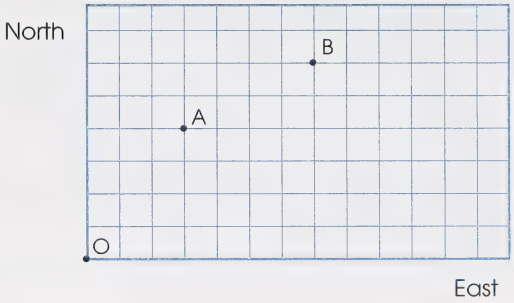
Check your answers by turning to the Appendix.

Did You Know?

René Descartes (1596–1650) was a French mathematician who developed the process of graphing relations using ordered pairs of numbers and describing these relations with equations.

As a boy Descartes had poor health. The rector of the Jesuit school Descartes attended often allowed him to stay in his room until he felt well enough to go to class. During those mornings in his room, Descartes often amused himself by watching a fly on the ceiling and got some ideas about the way the fly moved.

Suppose Descartes' ceiling looked like this.



If the fly started at O and moved to A, Descartes would say that A was at the point $(3E, 4N)$, meaning three spaces east of O and four spaces north of O. If the fly moved to B, Descartes would say that B was at the point $(7E, 6N)$, meaning seven spaces east of O and six spaces north of O.

Descartes tried to form equations that described the fly's path. He also made graphs of various equations.

For example, suppose Descartes wanted to graph the equation $E + N = 5$. He would make a table like this. He would then plot the points given on the table and connect the points.

East	North
0	5
1	4
2	3
3	2
4	1
5	0

Now Try This

Use a problem-solving strategy to answer the following question.

6. Adam drops a ball from a window 40 m above the sidewalk. The ball always bounces 0.5 times the height it drops. How high will the ball reach after each of the following bounces?

a. the first bounce
b. the second bounce
c. the third bounce
d. the fourth bounce

Check your answers by turning to the Appendix.

In this activity you used graphs to show relations. You extrapolated and interpolated from data to make predictions. You found a relation rule from a graph and used the rule to predict other values. You discovered some mathematical history and you continued to solve non-routine problems.

Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

In this section you described relations using arrow diagrams, tables, ordered pairs, relation rules, and graphs.

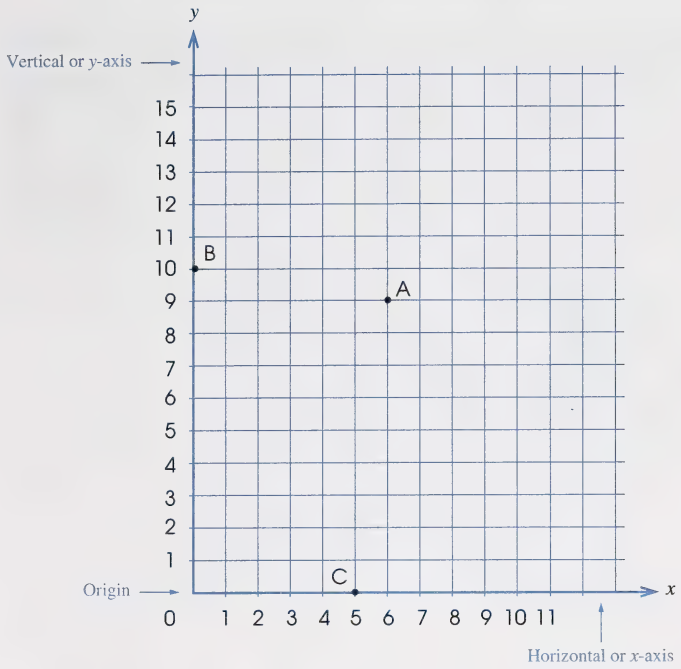
If you have difficulty reading graphs and drawing graphs, you may find this review helpful.

The **Cartesian coordinate system** is a way of defining the position of a point in two-dimensional space. There are two axis—the horizontal or ***x*-axis** and the vertical or ***y*-axis**.

The **coordinates of a point** are the numbers in the ordered pair. They show the position of a point with respect to the x-axis and the y-axis.

The **origin** is the point where the x-axis and the y-axis meet.

Example 1



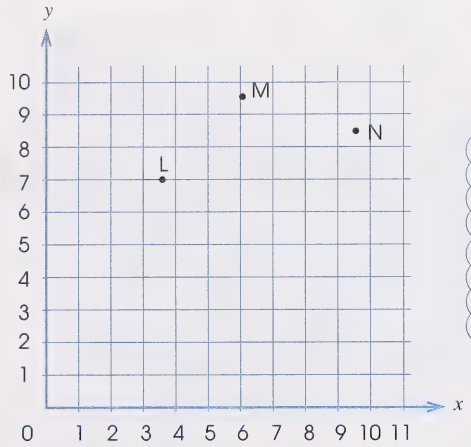
I get it! Point B has the coordinates (0, 10). This means that if I started at the origin, I would have to go 0 units along the x-axis and 10 units up to get to the point.

And Point C has the coordinates (5, 0). This means that if I started at the origin, I would have to go 5 units to the right along the x-axis and 0 units up to get to the point.

In Example 1, the coordinates of Point A are (6, 9). This means that if you started at the origin, you would have to go 6 units to the right along the x-axis and 9 units up to get to the point.

That's right! Remember too, that in an ordered pair such as (6, 9), the horizontal or x-coordinate always comes first. The vertical or y-coordinate always comes second.

Example 2



Notice that points L, M, and N are **not** located at the intersections of grid lines on this graph.

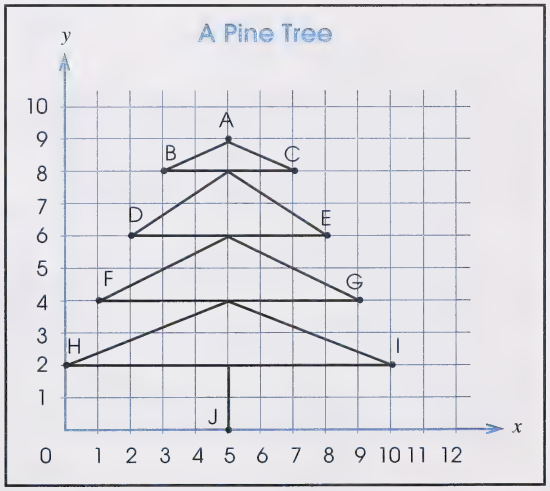
In this graph, the coordinates of Point L are about $\left(3\frac{1}{2}, 7\right)$. This means that if you started at the origin, you would have to go about $3\frac{1}{2}$ units to the right along the x-axis and 7 units up to get to the point.

So, the coordinates of Point M are about $\left(6, 9\frac{1}{2}\right)$. This means that if I started at the origin, I would have to go 6 units to the right along the x-axis and about $9\frac{1}{2}$ units up to get to the point.

And the coordinates of Point N are about $\left(9\frac{1}{2}, 8\frac{1}{2}\right)$. This means that if I started at the origin, I would have to go about $9\frac{1}{2}$ units to the right along the x-axis and about $8\frac{1}{2}$ units up to get to the point.

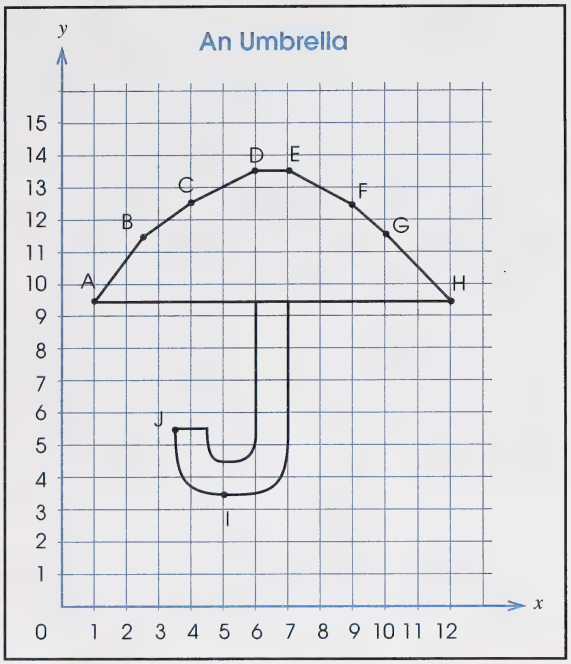
1. Make a table like this. Complete the table by giving the coordinates of the points on the pine tree graph.

Point	A	B	C	D	E	F	G	H	I	J
Coordinates										



2. Make a table like this. Complete the table by giving the coordinates of the points on the umbrella graph.

Point	A	B	C	D	E	F	G	H	I	J
Coordinates										



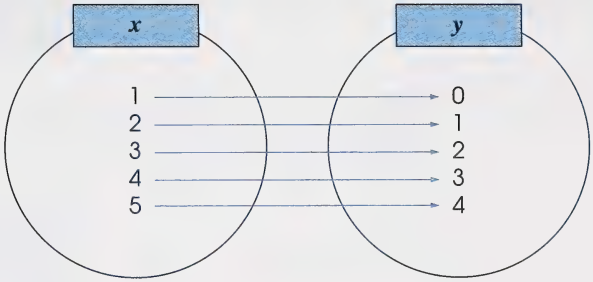
Check your answers by turning to the Appendix.

Enrichment

You have described patterns by writing relation rules. Another way to describe a relation is to write an equation.

Example

Write an equation to describe this relation.



Solution

In this arrow diagram, the first number is represented by the variable x. The second number is represented by the variable y.

$$x \longrightarrow y$$

For each pairing, the second number y is one less than the first number x.

$$x \longrightarrow x-1$$

So, the equation $y = x - 1$ describes the relation.

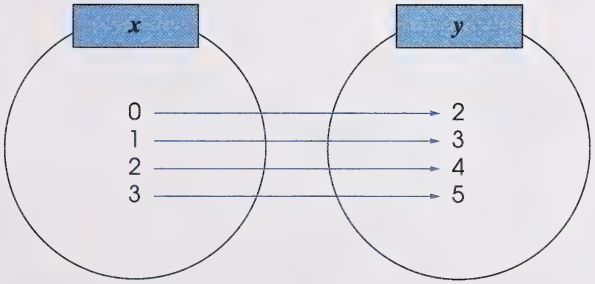
Note: The relationship between x and y in the example can be shown in the following table.

x	y	Pattern
1	0	$0 = 1 - 1$
2	1	$1 = 2 - 1$
3	2	$2 = 3 - 1$
4	3	$3 = 4 - 1$
5	4	$4 = 5 - 1$
...	...	
x	$x-1$	$y = x - 1$

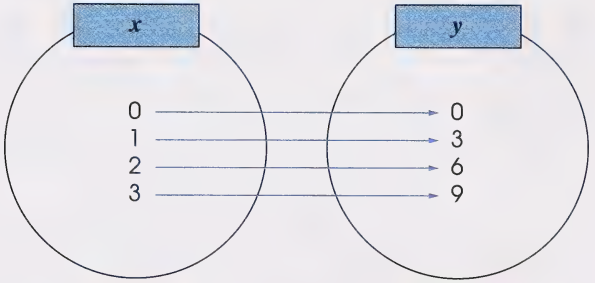
Notice that the value of the second number y depends on the value of the first number x.

1. Write an equation to describe each of the following arrow diagrams.

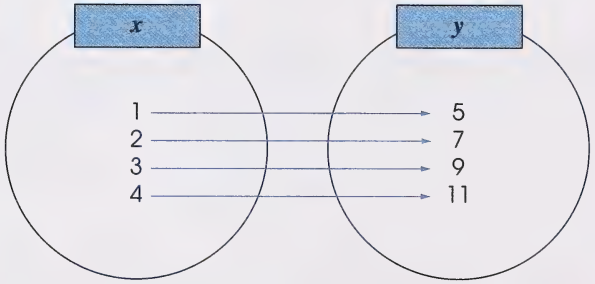
a.



b.



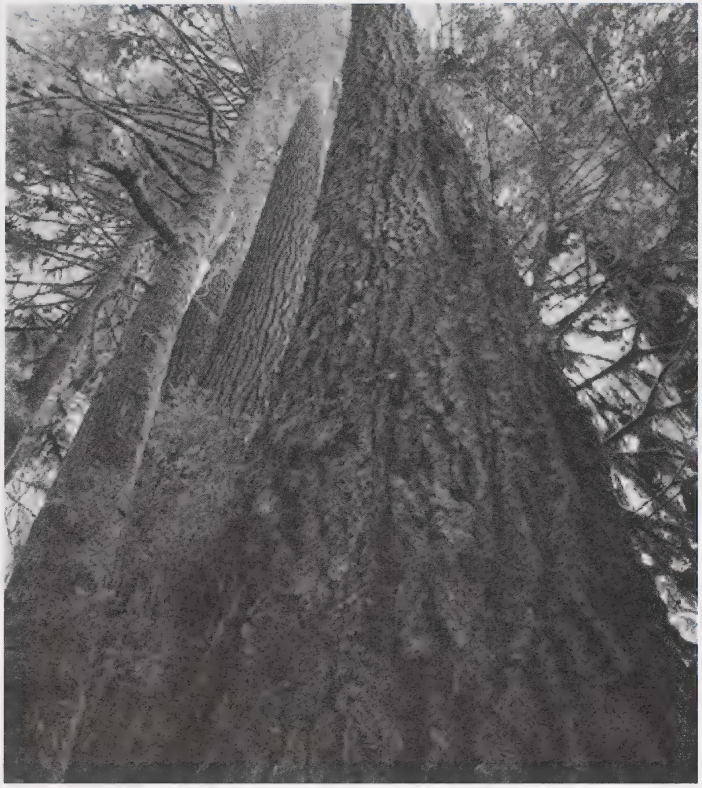
c.



2. Use each of the equations you determined in 1.a., 1.b., and 1.c. to find the value of the second number y if the value of $x = 10$.

Check your answers by turning to the Appendix.

Conclusion



In this section you examined patterns and relations. You described the relationship between two sets of numbers with arrow diagrams, ordered pairs, tables, rules, and graphs. You extrapolated and interpolated from data to make predictions. You found a relation rule from a graph and used the rule to predict other values.

You have discovered that the age of a tree is related to the number of rings. Do you think there is a relationship between the height of a tree and its age? Why or why not?

Assignment

Assignment Booklet

You are now ready to complete the module assignment for Section 2.

Section 3: From Measurement to Algebra

ALBERTA AGRICULTURE, CONSERVATION DEPARTMENT

How do you suppose that farmers find the area of a rectangular field that they intend to seed? Did you know that many farmers use algebra to find this measurement?

Mathematicians have discovered how the area of a rectangle is related to its length and width. They have used this relationship to write an equation called a formula. The area formula for a rectangle can be conveniently used to find the area when the length and width are known.

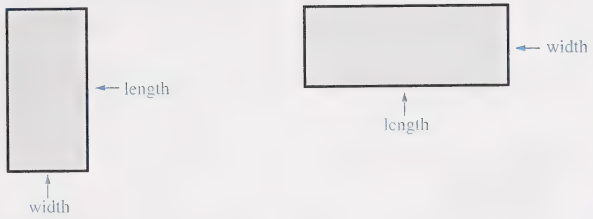
In this section you will discover and use formulas for the perimeter of a rectangle, the area of a rectangle, the volume of a rectangular solid, and the circumference of a circle. You will discover how geometry and algebra are related as you investigate rectangles, rectangular prisms, and circles.

Activity 1: Perimeter and Area of a Rectangle

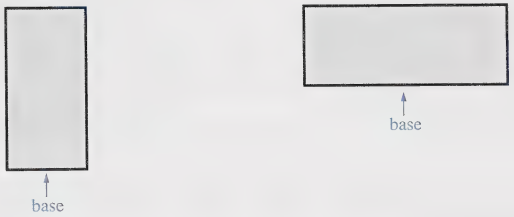
One of the most common shapes is the rectangle. Windows, doors, and carpets are usually rectangles. Gardens, tennis courts, and swimming pools are often rectangles.

In order to discuss rectangles, you will now review the names of different parts of a rectangle.

The longer side of a rectangle is usually called its **length**. The shorter side of a rectangle is called its **width**.



The bottom side of a rectangle (the side on which the rectangle rests) is also called the **base**.

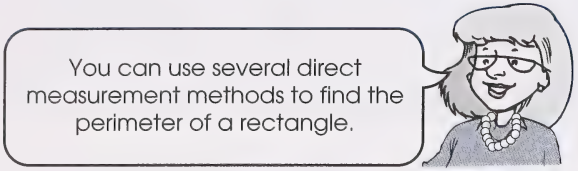


Sometimes the base is called the length, regardless if it is the longer or shorter side.

Perimeter

When you frame a picture, edge a rectangular garden with bricks, or fence a retangular dog run, you use **perimeter**.

The perimeter is the distance around a figure.



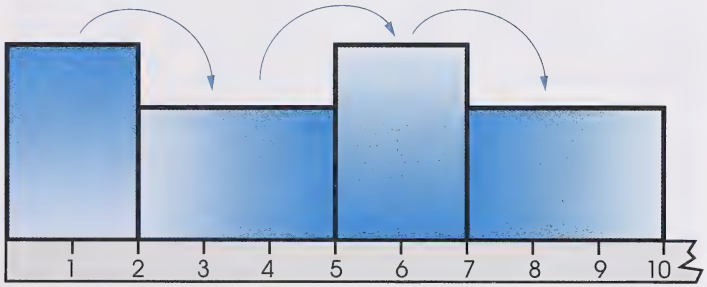
Example 1

Find the perimeter of a rectangle that is 3 units long and 2 units wide.

Solution

Method 1: "Rolling" the Rectangle

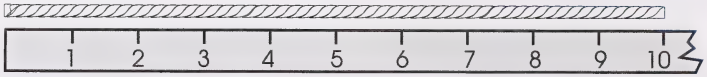
"Roll" the rectangle along a metric ruler or metre stick.



The perimeter of this rectangle is 10 units.

Method 2: Using String

Wind a string around the outside of the rectangle. Then measure the string.

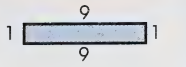
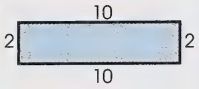
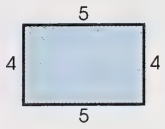
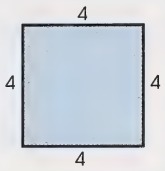


The perimeter of this rectangle is 10 units.

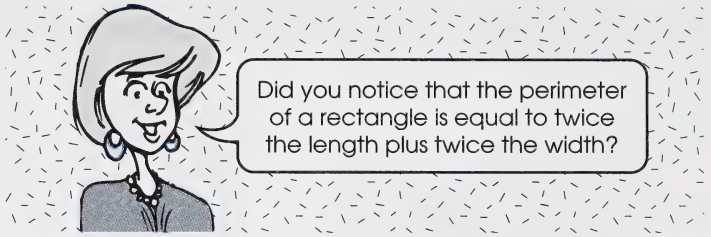
Photocopy the page of rectangles found in the Appendix, glue the page to heavy paper, and cut out the rectangles.

1. Use one of the direct measurement methods shown in Example 1 to find the perimeter of each of the rectangles that you cut out.

2. The following table gives the lengths, widths, and perimeters of various rectangles. What pattern do you notice? How is the perimeter of a rectangle related to the length and width of the rectangle?

Rectangle (measurement in units)	Length (units)	Width (units)	Perimeter (units)	
	9	1	20	9+9+1+1
	10	2	24	10+10+2+2
	5	4	18	5+5+4+4
	4	4	16	4+4+4+4

Check your answers by turning to the Appendix.



This relationship can be written as an equation.

$$P = 2\ell + 2w$$

The multiplication signs are not written.

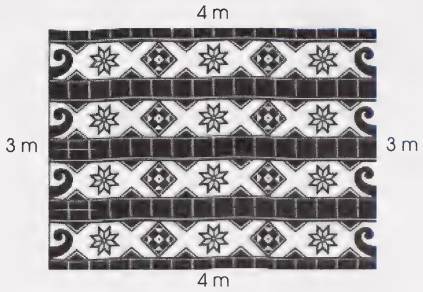
The equation is called a **formula**.

A formula is an equation that can be used to find the value of a variable when you are given the value of the other related variable or variables.

The formula allows you to calculate the perimeter of a rectangle when you are given the length and width.

Example 2

Find the perimeter of this carpet.



Solution

$P = 2\ell + 2w$

$= (2 \times 4) + (2 \times 3)$

$= 8 + 6$

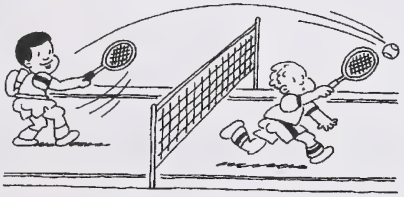
$= 14$

Follow the rules for the order of operations.

The perimeter of the rug is 14 m.

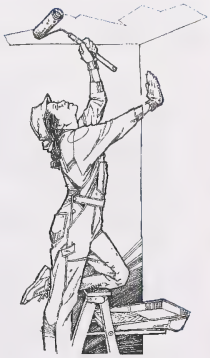
3. Tom wants to put molding around the perimeter of the ceiling he is constructing. The rectangular room is 4.5 m by 6 m. How much molding will he need?
Hint: Assume there is no waste.

4. A tennis court is 23.8 m by 11 m. What is the perimeter of the tennis court?



Check your answers by turning to the Appendix.

Area



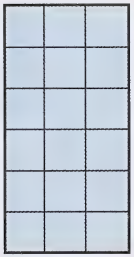
When you paint a ceiling, wallpaper a wall, cover a floor with carpet, or plant a field, you use **area**.

Area is the amount of surface a figure covers. It is expressed in square units.

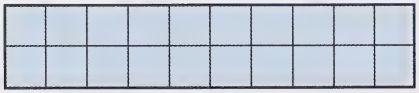
You can find the area of a rectangle by counting the number of square units.

5. Find the area in square units of each rectangle.

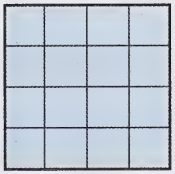
a.



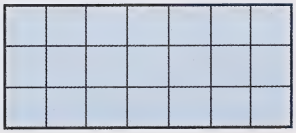
b.



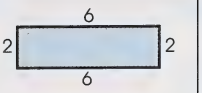
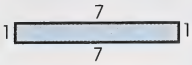
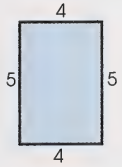
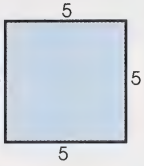
c.



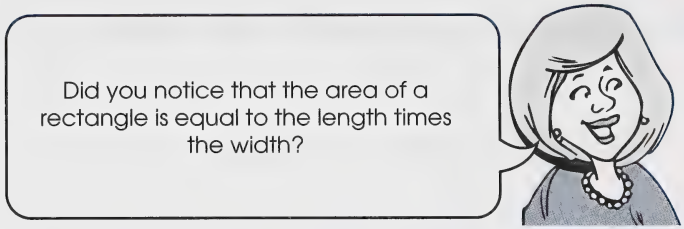
d.



6. The following table gives the lengths, widths, and areas of various rectangles. What pattern do you notice? How is the area of a rectangle related to the length and width of the rectangle?

Rectangle (measurement in units)	Length (units)	Width (units)	Area (square units)	
	6	2	12	6×2
	7	1	7	7×1
	5	4	20	5×4
	5	5	25	5×5

Check your answers by turning to the Appendix.



This relationship can be expressed as a formula.

$A = \ell w$

The multiplication sign is not written.

This formula allows you to calculate the area of a rectangle when you are given the length and width.

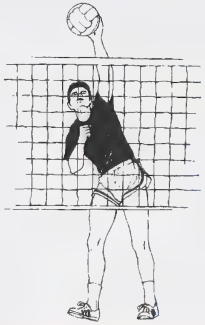
Example

Find the area of a volleyball court which is 9 m by 18 m.

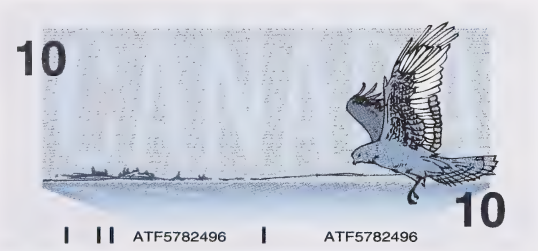
Solution

$$A = \ell w$$
$$= 18 \times 9$$
$$= 162$$

The area is 162 m^2.



7. Find the area of a Canadian $10 bill whose length is 15.2 cm and width is 7 cm.



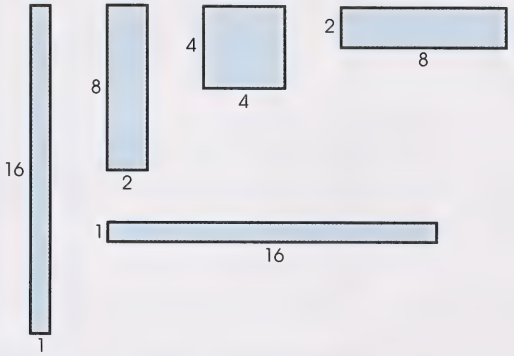
8. A Canadian flag is 3 m by 1.5 m. Find its area.

Check your answers by turning to the Appendix.

Relating Perimeter and Area to Length

In this part of the activity you will discover how the perimeter and area of a rectangle are related to the length of the rectangle. You will also use graphs to discover more about how the perimeter and area of a rectangle are related.

9. There are many rectangles with an area of 16 square units. Here are some of the possibilities.



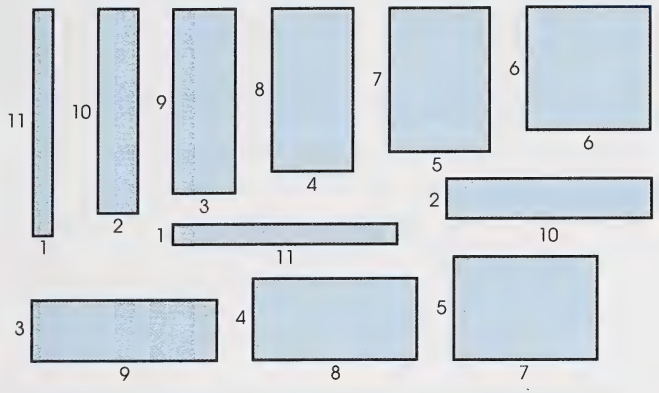
a. Use the given rectangles to complete a table like this.

Rectangles with an Area of 16 Square Units		
Length (units)	Width (units)	Perimeter (units)
1	16	
2	8	
4	4	
8	2	
16	1	

b. Give the rectangle or rectangles with the greatest perimeter.

c. Give the rectangle or rectangles with the least perimeter.

10. There are many rectangles with a perimeter of 24 units. Here are some of the possibilities.



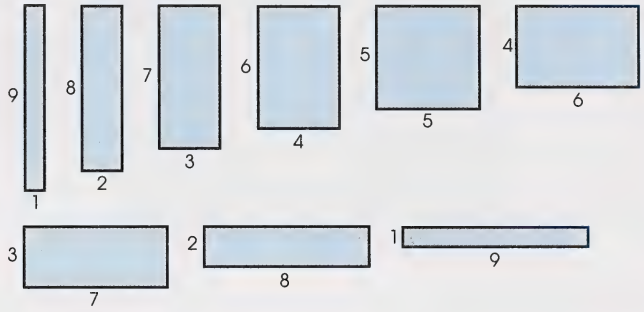
a. Use the given rectangles to complete a table like this.

Rectangles with a Perimeter of 24 Units		
Length (units)	Width (units)	Area (square units)
1	11	
2	10	
3	9	
4	8	
5	7	
6	6	
7	5	
8	4	
9	3	
10	2	
11	1	

b. Give the rectangle or rectangles with the greatest area.

c. Give the rectangle or rectangles with the least area.

11. There are many rectangles with a perimeter of 20 units. Here are some of the possiblilities.



a. Use the given rectangles to complete a table like this.

Rectangles with a Perimeter of 20 Units		
Length (units)	Width (units)	Area (square units)
1	9	
2	8	
3	7	
4	6	
5	5	
6	4	
7	3	
8	2	
9	1	

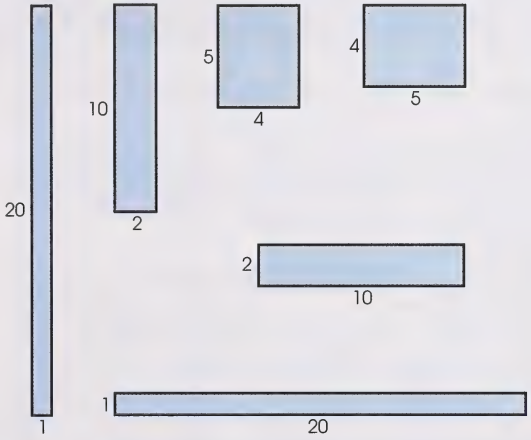
b. Graph the set of ordered pairs, (length, area), for the rectangles with a perimeter of 20 units.

c. Because a rectangle with a perimeter of 20 units can have fractional units for its length, connect the dots on the graph in 11.b.

d. What does the graph tell you about the relationship between length and area?

e. What do you notice about the rectangle with the greatest area?

12. There are many rectangles with an area of 20 square units. Here are some possibilities.



a. Use the given rectangles to complete a table like this.

Rectangles with an Area of 20 Square Units		
Length (units)	**Width (units)**	**Perimeter (units)**
1	20	
2	10	
4	5	
5	4	
10	2	
20	1	

b. Graph the set of ordered pairs, (length, perimeter), for the rectangles with an area of 20 square units.

c. Because a rectangle with an area of 20 square units can have fractional units for its length, connect the dots on your graph in 12.b.

d. What does the graph tell you about the relationship between length and perimeter?

Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following questions. Use only whole-number lengths to solve these problems.

13. Arlene wants to build a rectangular pen for her chickens. The chickens need an area of 36 m^2. To save on fencing, she plans to use a river that runs through her farm as one side of the pen.

a. Draw and label all the possible designs for the pen.
b. Which is the most economical design for the pen?
c. Which is the most expensive design for the pens?

14. Optek decides to build a rectangular dog run for his Samoyed. He wants to use as little fencing as possible. If the dog needs about 144 m^2, how long and wide should the dog run be?

15. A hardware store sells fencing by the metre. The store has 42 m of fencing left. Mr. Petruk purchases all of the fencing to make a rectangular pen. What will the dimensions of the pen be if the length is 1 m more than the width?

Check your answers by turning to the Appendix.

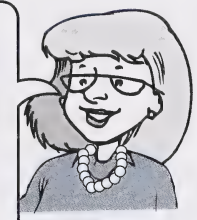
Use a problem-solving strategy to answer the following question.

16. Kyla, Tom, Barb, Sharon, and Mario are friends. Kyla is taller than Mario and she is also taller than Sharon. Sharon is taller than Tom but shorter than Mario. Barb is the second tallest of the friends. Order the friends by height (tallest to shortest).

JIM WHITMER PHOTOGRAPHY

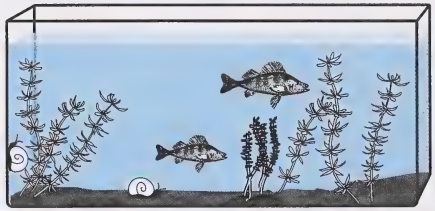
Check your answers by turning to the Appendix.

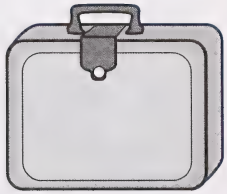
In this activity you discovered the formulas for the perimeter of a rectangle and the area of a rectangle. You used these formulas to calculate the perimeter and area of a rectangle. You discovered how the perimeter and area of a rectangle are related and you showed this relationship on a graph. You continued to solve non-routine problems.



Activity 2: Volume of a Rectangular Prism

When you fill an aquarium or pack a suitcase you use **volume**.





Volume is a measurement of how much space an object occupies. Volume is expressed in cubic units.

You can find the volume of a rectangular prism by counting the number of cubic units.

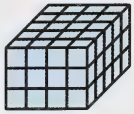
Example 1

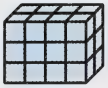
The volume of this rectangular prism is 18 cubic units.

Some of the small cubes are hidden.

1. Find the volume in cubic units of each of the following rectangular prisms. If you wish, you can first build the rectangular prisms with sugar cubes or blocks.

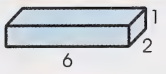
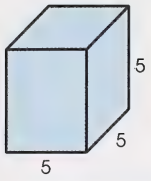
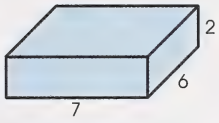
a.

b. 

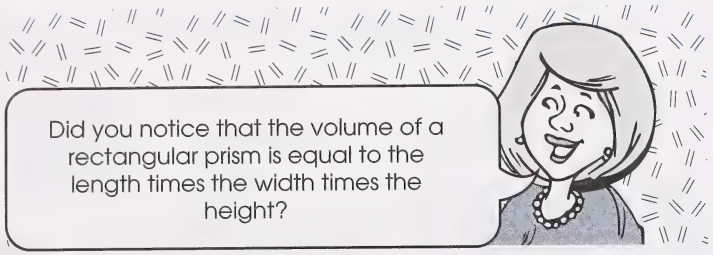
c. 

d.

2. The following table gives the lengths, widths, heights, and volumes of various rectangular prisms. What pattern do you notice? How is the volume of a rectangular prism related to the length, width, and height?

Rectangular Prisms (measurement in units)	Length (units)	Width (units)	Height (units)	Volume (cubic units)	
	6	2	1	12	← $6 \times 2 \times 1$
	5	5	5	125	← $5 \times 5 \times 5$
	7	6	2	84	← $7 \times 6 \times 2$

Check your answers by turning to the Appendix.



This relationship can be expressed as a formula.

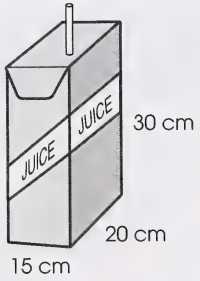
$V = \ell wh$

The multiplication signs are not written.

This formula allows you to calculate the volume of a rectangular prism when you are given the length, width, and height.

Example 2

Find the volume of a juice container that is 15 cm by 20 cm by 30 cm.



Solution

$V = \ell wh$

$= 15 \times 20 \times 30$

$= 9000$

The volume is $9000\ \text{cm}^3$.

3. Find the volume of a small cassette recorder that is 2 cm by 15 cm by 7 cm.

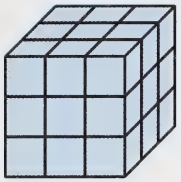
4. The cargo space of Lauri's truck is 2.2 m by 2.4 m by 5.5 m. What is the volume of the cargo space? Round the answer to the nearest tenth.

Check your answers by turning to the Appendix.

Now Try This

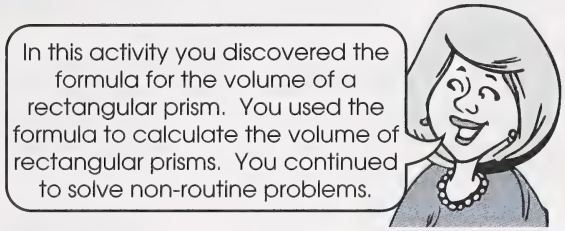
Use a problem-solving strategy to answer the following question.

5. You are given a cube that is 3 units by 3 units by 3 units. The cube is painted and then cut up into 27 cubes that are 1 unit by 1 unit by 1 unit. Some of these small cubes will have three faces painted, some will have two faces painted, some will have one face painted, and some will have no paint on them at all.



a. How many small cubes will have three faces painted?
b. How many small cubes will have two faces painted?
c. How many small cubes will have one face painted?
d. How many small cubes will have no paint?

Check your answers by turning to the Appendix.


In this activity you discovered the formula for the volume of a rectangular prism. You used the formula to calculate the volume of rectangular prisms. You continued to solve non-routine problems.

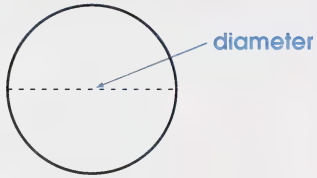
Activity 3: Circumference of a Circle

Have you ever assisted your parents or friends with car or plumbing repairs?

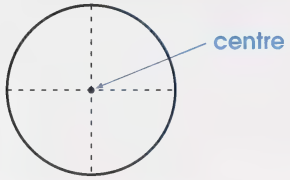
Working with pipes, hoses, nuts, and bolts requires a knowledge about the various parts of a circle.

Each circle has a **diameter** and a **centre**.

If you cut out a circle and fold it in half, the line segment formed is called the diameter of the circle.

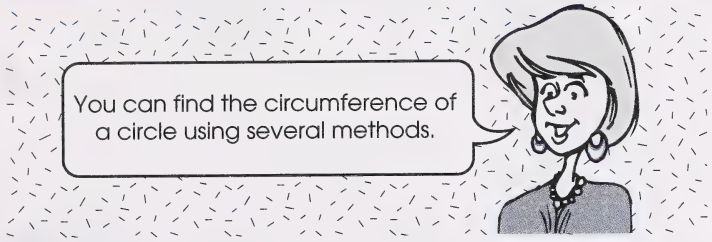


If you fold the circle in half again, another diameter is formed. The point where the two diameters cross is called the centre of the circle.



The circumference of this circle is about 6 units.

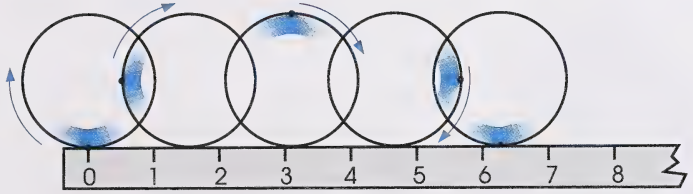
The distance around a circle is called the **circumference**.



Example 1

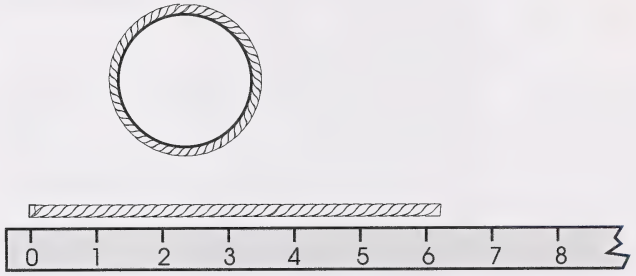
Method 1: Rolling the Circle

Mark the circle and roll it along a metric ruler or metric stick until the mark touches the ruler again.



Method 2: Using String

Wind a string around the outside of the circle. Then measure the string.



The circumference of this circle is about 6 units.

Photocopy the page of circles found in the Appendix, glue the page to heavy paper and cut out the circles.

1. Use one of the direct measurement methods shown in Example 1 to find the circumference of each of the circles that you cut out.
   Note: Keep the circles; you will need them in question 2.

Use a calculator to answer question 2.

2. a. Complete a table like the following.

For each of the circles you cut out, express the ratio of the circumference to the diameter as a decimal number rounded to the nearest hundredth.

Circle	Circumference (cm)	Diameter (cm)	Ratio of Circumference to the Diameter
Circle A			
Circle B			
Circle C			
Circle D			
Circle E			

b. What do you notice about the ratio of the circumference of each circle to the diameter of each circle?

Check your answers by turning to the Appendix.

Did you notice that regardless of the size of the circle, the ratio of the circumference to the diameter is between 3 and 4?

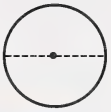
This ratio is called pi, and it is represented by the Greek letter π. π is the first letter of the Greek word "περιμετρος" meaning "perimeter."

π is not a variable. It represents a constant number that is between 3 and 4. Usually π is rounded to the nearest hundredth.

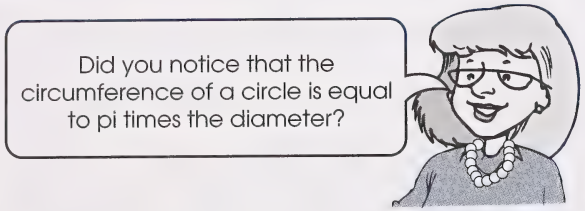
$\pi \doteq 3.14$

You can find out more about π in the Enrichment.

3. The following table gives the diameters and circumferences of various circles. What pattern do you notice? How is the circumference of a circle related to the diameter?

Circles (measurements in units)	Diameter (units)	Circumference (units)	
	12.0	37.7	about 3.14×12.0
	18.0	56.5	about 3.14×18.0
	20.0	62.8	about 3.14×20.0

Check your answers by turning to the Appendix.



This relationship can be written as a formula.

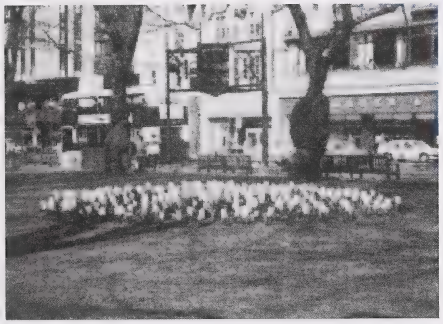
$C = \pi d$

The multiplication sign is not written.

This formula allows you to find the circumference of a circle when you are given the diameter.

Example 2

What is the circumference of a circular flower garden with a diameter of 4.2 m?

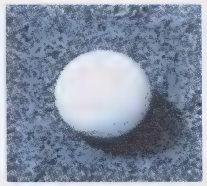


Solution

$$C = \pi d$$
$$\doteq 3.14 \times 4.2$$
$$\doteq 13.2$$

The circumference of the garden is about 13.2 m.

4. A golf ball has a diameter of 4.27 cm. What is the circumference of the golf ball? Round the answer to the nearest hundredth.



5. The diameters of the three larger rings on a sheet of curling ice are 3.66 m, 2.44 m, and 1.22 m. Calculate the circumference of each ring. Round each answer to the nearest hundredth.

Now Try This

Use a problem-solving strategy to answer the following question.

6. Noelle noted the jewellery that a group of 20 women wore. Of the group, 0.75 wore a necklace, 0.8 wore a watch, and 0.1 wore neither a watch nor a necklace.

a. How many women wore only a watch for jewellery?

b. how many women wore only a necklace for jewellery?

c. How many women wore both a watch and a necklace?

Check your answers by turning to the Appendix.

In this activity you discovered that the ratio of the circumference of a circle to the diameter is a constant number between 3 and 4. This number is represented by the Greek letter π, and is approximately 3.14. You discovered and used a formula for the circumference of a circle. You continued to solve non-routine problems.

Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

In this section you developed formulas for the perimeter of a rectangle, area of a rectangle, volume of a rectangular prism, and circumference of a circle. These formulas allow you to calculate certain measurements indirectly if you are given the other measurements.

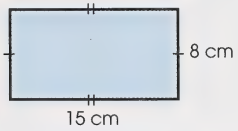
You may find it helpful to review these formulas.

Finding the Perimeter of a Rectangle

The opposite sides of rectangles are equal. This property helps you to calculate the perimeter when you are given the measurements of only two sides.

Example

Find the perimeter of the given rectangle.



The slash marks indicate which sides are equal.

Solution

Method 1: Using Addition

You can find the perimeter of any rectangle by adding the sides.

$$\begin{aligned} P &= s_1 + s_2 + s_3 + s_4 \\ &= 15 + 15 + 8 + 8 \\ &= 46 \end{aligned}$$

S_1 means the first side, S_2 means the second side,....

The perimeter of the rectangle is 46 cm.

Method 2: Using Multiplication and Addition

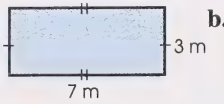
You can find the perimeter of any rectangle by doubling the length and the width, and then adding the products.

$$\begin{aligned} P &= 2\ell + 2w \\ &= 2\times 15 + 2\times 8 \\ &= 30 + 16 \\ &= 46 \end{aligned}$$

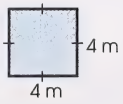
The perimeter of the rectangle is 46 cm.

1. Calculate the perimeter of each of the following rectangles.

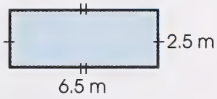
a.



b.



c.



Check your answers by turning to the Appendix.

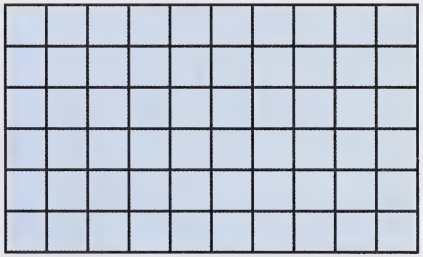
Finding the Area of a Rectangle

Area is the number of square units in a figure.

You do not need to count all the square units to find the area of a rectangle. You can simply count the number of square units in a row and multiply that number by the number of rows.

Example

Find the area of the given rectangle.



Solution

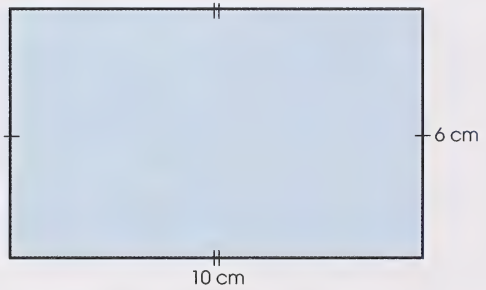
In the given rectangle, there are ten square units in one row. There are six rows.

$$A = 10 \times 6$$
$$= 60$$

So, the area of the rectangle is 60 square units.

Note: If the square unit in the example is 1 cm^2, then the area of the rectangle is 60 cm^2.

If the rectangle was not divided into square units, you could still find the area if you knew the dimensions of the rectangle.



The number of square units in one row corresponds to the length of the base of the rectangle. The number of rows corresponds to the width of the rectangle. This relationship can be expressed by the following formula.

$$A = \ell w$$

You can use this formula to find the area of the rectangle indirectly.

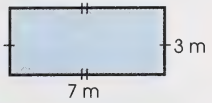
$A = \ell w$
$= 10 \times 6$
$= 60$

The length of the rectangle is 10 cm.
The width of the rectangle is 6 cm.

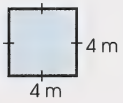
So, the area of the rectangle is 60 cm^2.

2. Calculate the area of each of the following rectangles.

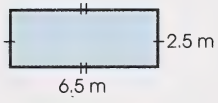
a.



b.



c.



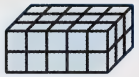
Check your answers by turning to the Appendix.

Finding the Volume of a Rectangular Prism

Volume is the number of cubic units. You do not need to count all the cubic units to find the volume of a rectangular prism. You can simply count the number of units in a row and multiply by the number of rows to find the number of cubic units in a layer. Then multiply by the number of layers.

Example

Find the volume of this rectangular prism.



Solution

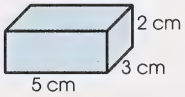
Each layer in the prism has 5 rows of 3 units.
There are 2 layers.

$V = 5 \times 3 \times 2$
$= 30$

So, the volume of the rectangular prism is 30 cubic units.

Note: If the cubic unit in the example is 1 cm^3, then the volume of the rectangular prism is 30 cm^3.

If the rectangular prism in the example was not divided into cubic units, you could still find the volume if you knew the dimensions of the rectangle.



The number of cubic units in one layer corresponds to the length times the width. The number of layers corresponds to the height. So, the volume or total number of cubic units is the same as the length times the width times the height. This can be expressed by the following formula.

$V = \ell wh$

You can use this formula to find the volume of the rectangular prism indirectly.

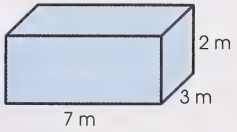
$$V = \ell wh$$
$$= 5 \times 3 \times 2$$
$$= 30$$

The length is 5 cm.
The width is 3 cm.
The height is 2 cm.

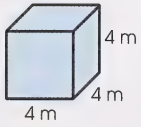
So, the volume of the rectangular prism is 30 cm^3.

3. Calculate the volume of each of the following rectangular prisms.

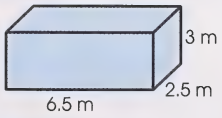
a.



b.



c.

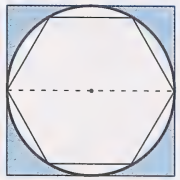


Check your answers by turning to the Appendix.

Finding the Circumference of a Circle

The distance around a circle is the circumference of a circle.

The given diagram shows a square, a circle, and a hexagon. Each side of the square is 2 cm, the diameter of the circle is 2 cm, and each side of the hexagon is 1 cm.



You can estimate the circumference of the circle by using this reasoning.

- The circle has a circumference that is less than the perimeter of the square and greater than the perimeter of the hexagon.
- The perimeter of the square is 8 cm.

$$2 + 2 + 2 + 2 = 8$$

- The perimeter of the hexagon is 6 cm.

$$1 + 1 + 1 + 1 + 1 + 1 = 6$$

So, the circumference of the circle is between 6 cm and 8 cm.

You can check your estimate by using the circumference formula.

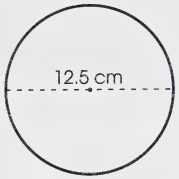
$$C = \pi d$$
$$\doteq 3.14 \times 2$$
$$\doteq 6.28$$

The circumference of the circle is about 6.28 cm. So, your estimate is correct.

4. Calculate the circumference of each of the following circles.

a.

b.



Check your answers by turning to the Appendix.

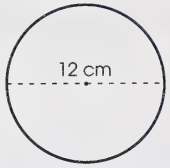
Enrichment

You may not at first notice the π key on your calculator. Examine your scientific calculator and find the EXP key; the symbol π will probably appear on the calculator just above this key. To use the π key, press the coloured INV **or** 2nd F key and then press the EXP key.

You may use a scientific calculator to find the circumference of a circle. Scientific calulators have a π key.

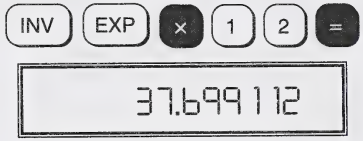
Example

Find the circumference of this circle. Round the answer to the nearest whole number.



Solution

Use the formula $C = \pi d$.

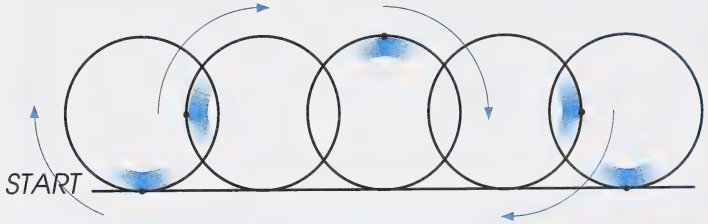


Note: The 2nd F key may be used instead of the INV key.

So, the circumference is about 38 cm.

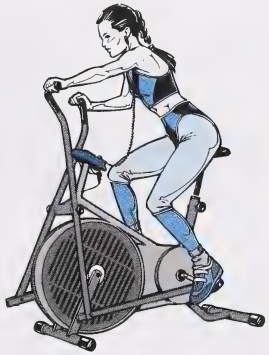
Use a calculator to answer questions 1 to 3.

1. A bicycle wheel has a diameter of 66 cm. How far does the bicycle travel when the tire makes one complete rotation? Round the answer to the nearest whole number.



2. The diameter of the tire of William's tractor is 1.5 m. How far does the tractor travel when the tire makes one complete rotation? Round the answer to the nearest tenth.

3. The wheel on Kay's stationary exercise bike has a diameter of 40 cm. An odometer on the bike records the distance "travelled." How many times would the wheel turn for the odometer to show a distance of 1 km? Round the answer to the nearest whole number.



Check your answers by turning to the Appendix.

Now Try This

Over the years mathematicians have used many fractions for π. The following table lists a few examples.

Date	Fractions Used for π
Around 2000 B.C.	The Babylonians use $\frac{25}{8}$.
	The Egyptians use $\frac{256}{81}$.
212 B.C.	Archimedes, a Greek mathematician, uses $\frac{22}{7}$.
A.D. 150	Ptolemy of Alexandria uses $\frac{377}{120}$.
A.D. 480	Tsu Chung-chi, a Chinese mathematician, uses $\frac{355}{113}$.
A.D. 530	Aryabhata, a Hindu mathematician, uses $\frac{62832}{20000}$.
A.D. 1150	Bhaskara, a Hindu mathematician, uses $\frac{3297}{1250}$.
A.D. 1220	Fibonacci, an Italian mathematician, uses $\frac{864}{274}$.

In 1767 Lambert, a Swiss-German mathematician, proved that π is an irrational number. This means that there is no fractional or decimal number that exactly represents π. Moreover, the digits do not have a repeating pattern and go on indefinitely.

Today mathematicians using computers have calculated the value of π to millions of decimal places. The value of π to the first nine decimal places is 3.141 592 653.

At the time this course was printed, there were several sites about pi on the Internet. Here is the uniform resource locator (URL) of one site.

http://www.ccsf.caltech.edu/~roy/pi.html

Click on "Digits" to discover the first 50 000 digits of pi.

Use a calculator to answer question 4.

4. Each of the following fractions has been used for π. Change each to a decimal number.

a. $\frac{25}{8}$ **b.** $\frac{256}{81}$ **c.** $\frac{377}{120}$

d. $\frac{22}{7}$ **e.** $\frac{355}{113}$ **f.** $\frac{62832}{20000}$

g. $\frac{3927}{1250}$ **h.** $\frac{864}{274}$

5. To how many decimal places is each of the fractions in question 4 accurate? (Compare the decimal numbers you calculated in question 4 to 3.141 592 6.)

Check your answers by turning to the Appendix.

Did You Know?

About 1650, John Wallis an English mathematician, showed that

$$\frac{\pi}{2} = \frac{2\times2\times4\times4\times6\times6\times8\times8}{1\times3\times3\times5\times5\times7\times7\times9}\cdots$$

Conclusion

In this section you developed formulas for the perimeter of a rectangle, the area of a rectangle, the volume of a retangular prism, and the circumference of a circle. You used these formulas to calculate the perimeter and area of a rectangle given the length and width; the volume of a rectangular prism given the length, width, and height; and the circumference of a circle given the diameter.

Farmers are able to use many of these algebraic formulas to calculate the area of a field they intend to seed, the amount of fencing they may require to enclose a pasture, or the volume of grain they are shipping to market.

Assignment

Assignment Booklet

You are now ready to complete the module assignment for Section 3.

Module Summary

R.C.M.P

In this module you investigated how algebra is related to arithmetic. You discovered the meaning of a variable and used variables to write variable expressions and conditional equations. You evaluated variable expressions given values of the variable, and you solved conditional equations. You used algebra to describe patterns and develop formulas.

You discovered that everybody uses algebra. Families need algebra to find the perimeter of a room when they are buying new baseboards and to find the area of a livingroom floor when they are installing new carpets. Business people need algebra when they use spreadsheets. Economists need algebra to show economic trends and make predictions. Police officers use algebra to calculate the braking distance at various speeds.

Final Module Assignment

Assignment Booklet

You are now ready to complete the final module assignment.

APPENDIX

- Glossary
- Suggested Answers
- Cut-out Learning Aids

Glossary

Algebraic equations: equations true only for certain values of the variable; also called **conditional equations**

Algebraic expression: see **variable expression**

Area: the amount of surface a figure covers expressed in square units

Arrow diagram: a diagram that shows the relation between two sets

Base: the bottom side of a rectangle; the side on which the rectangle rests

Cartesian coordinate system: a way of defining the position of a point in two-dimensional space

Centre: (of a circle) the point at which two diameters cross in a circle

Circumference: (of a circle) the perimeter or distance around a circle

Conditional equations: see **algebraic equations**

Consecutive numbers: numbers written in order

Coordinates: (of a point) the numbers in the ordered pair that show the position of a point on a grid

Diameter: the distance across a circle measured through the centre

Equation: any mathematical statement that contains an equal sign

Evaluating: finding the value of a mathematical expression

Extrapolation: the process of finding a value by following a pattern and going beyond the values already known

Formula: an equation that can be used to find the value of a variable when the value of the other related variable or variables are given

Inspection method: solving algebraic equations by looking at them

Interpolation: the process of finding a value between two other known values

Length: the name generally given to the longer side of a rectangle

Mapped onto: paired with

Mapping: a method of showing a relationship between pairs

Mathematical expression: a phrase that involves numbers and operations

Mersenne primes: the prime numbers 3, 7, 31, 127, 8191,..., $2^n - 1$, where n is a prime

Ordered pairs: a pair for which the order is important

Order of operations: a set of rules that mathematicians have agreed upon to find the value of a mathematical expression

Origin: the point at which the x-axis and the y-axis meet on a grid

Perimeter: the distance around a figure

Powers: a compact way of writing the products of equal factors

Problem: a task for which the method of finding the answer (as well as the answer) is not immediately known

Relation: a set of ordered pairs

Sequence: a set of numbers that is ordered according to some rule

Solution: the value of the variable that makes an equation true

Solving an equation: finding the value of a variable that makes the equation true

Terms: the numbers or items in a sequence

Variable: an unspecified number

Variable expression: a mathematical expression that uses one or more variables; also called an **algebraic expression**

Verification: the check to decide if an answer is the solution

Verifying the solution: checking the solution

Volume: a measurement of how much space an object occupies; measured in cubic units

Width: the name generally given to the shorter side of a rectangle

***x*-axis:** the horizontal axis on a graph

***y*-axis:** the vertical axis on a graph

Suggested Answers

Section 1: Activity 1

1. a. $4 + 5 \times 3$
$= 4 + 15$
$= 19$

b. $9 - 4 \times 2$
$= 9 - 8$
$= 1$

c. $8 \div 4 + 2$
$= 2 + 2$
$= 4$

d. $10 - 3 \div 3$
$= 10 - 1$
$= 9$

e. $8 \times 3 - 2 \times 4$
$= 24 - 8$
$= 16$

f. $6 + 5 \times 7 + 4$
$= 6 + 35 + 4$
$= 41 + 4$
$= 45$

g. $9 \div 3 \times 2 + 8$
$= 3 \times 2 + 8$
$= 6 + 8$
$= 14$

h. $2 \times 8 - 3 \times 4$
$= 16 - 12$
$= 4$

i. $9 \times 6 + 3 + 5 \times 6$
$= 54 + 3 + 30$
$= 57 + 30$
$= 87$

2. a. $36 \div 4 - 2^2$

$= 36 \div 4 - 2 \times 2$

$= 36 \div 4 - 4$

$= 9 - 4$

$= 5$

b. $8 - 3 + 2^3$

$= 8 - 3 + 2 \times 2 \times 2$

$= 8 - 3 + 8$

$= 5 + 8$

$= 13$

c. $3^2 + 5 \times 6$

$= 3 \times 3 + 5 \times 6$

$= 9 + 5 \times 6$

$= 9 + 30$

$= 39$

d. $2^3 + 3^3 \times 2$

$= 2 \times 2 \times 2 + 3 \times 3 \times 3 \times 2$

$= 8 + 27 \times 2$

$= 8 + 54$

$= 62$

3. a. $(4 + 2) \times 7$

$= 6 \times 7$

$= 42$

b. $(12 + 3) \div 3$

$= 15 \div 3$

$= 5$

c. $5 \times (6 - 2) + 3$

$= 5 \times 4 + 3$

$= 20 + 3$

$= 23$

d. $3^2 - (4 \times 2)$

$= 3^2 - 8$

$= 3 \times 3 - 8$

$= 9 - 8$

$= 1$

e. $4^2 - 2 \times (5 + 3)$

$= 4^2 - 2 \times 8$

$= 4 \times 4 - 2 \times 8$

$= 16 - 2 \times 8$

$= 16 - 16$

$= 0$

f. $(18 - 6) \div 4$

$= 12 \div 4$

$= 3$

4. a. 5 7 − 1 2 × 3 =

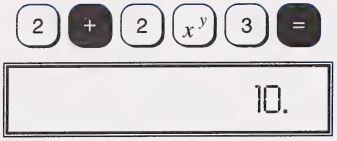
b. 7 + 0 • 8 ÷ 0 • 5 =

c. 3 • 2 + 1 6 • 4 ÷ 4 =

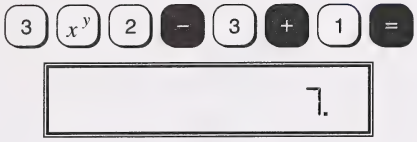
5. Note: Some scientific calculators have a y^x key instead of a x^y key.

a. 2 × 3 x^y 2 =

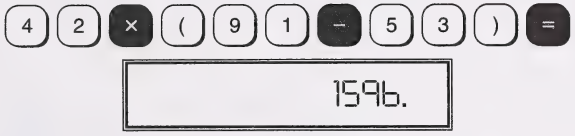
b.



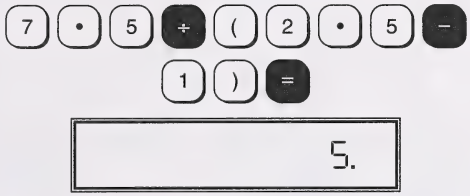
c.



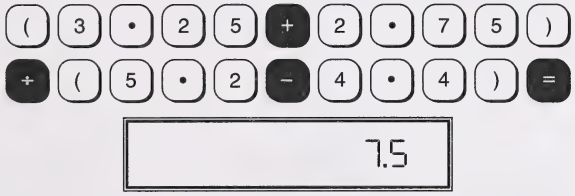
6. a.



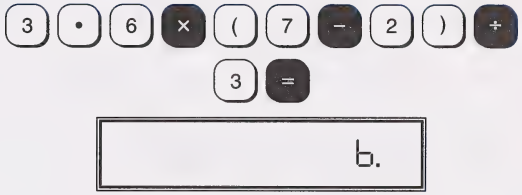
b.



c.



d.



7. Answers will vary. How many different ways were you able to list?

a. six plus seven
six increased by seven
seven more than six
seven added to six
the sum of six and seven

b. four minus one
four diminished by one
four decreased by one
four take away one
four subtract one
one less than four
one subtracted from four

c. two times five
two multiplied by five
the product of two and five

d. four divided by three

8. **a.** $9-7$ **b.** $6-4$ **c.** 5×8 **d.** $5+2$
e. $7+6$ **f.** $10-9$ **g.** $12\div3$ **h.** $5+2$

9. Answers may vary, but the meaning should be the same.

a. five less than the product of three and six
b. three, times six minus five
c. six minus two, plus one
d. six, minus two plus one
e. three more than the product of four and five
f. four times the sum of five and three
g. three less than the product of two and six
h. two, times six minus three
i. four, plus six divided by two
j. the sum of four and six, divided by two

10. **a.** $2\times3-8$ **b.** $12-8\div2$ **c.** $3\times5+2$
d. $3\times(5+8)$ **e.** $(5-3)\div2$

Now Try This

11. You may find it helpful to use objects such as a dollar coin, a quarter, and a dime to represent the people. Let the dollar coin represent the 125-kg person. The quarter will represent the 75-kg person, and the dime will represent the 50-kg person.

Note: The 125-kg person cannot travel in the canoe with anyone else.

Here is one possible solution. There are other possible solutions.

Trip 1: The 50-kg person and the 75-kg person paddle together across the river.

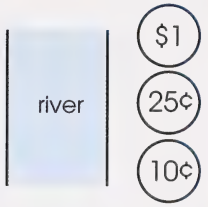
Trip 2: The 50-kg person stays on the other side. The 75-kg person paddles back across the river.

Trip 3: The 75-kg person gets out and the 125-kg person paddles across the river.

Trip 4: The 125-kg person gets out and the 50-kg person paddles back across the river.

Trip 5: The 50-kg person and 75-kg person paddle together across the river.

It takes five trips for the three people to get across the river.



Section 1: Activity 2

1. **Note:** Your choice of variable may be different.

a. $a+2$ **b.** $2d$ **c.** n^2 **d.** $5p$

e. $n-2$ **f.** $a+12$ **g.** $\frac{e}{2}$ or $e \div 2$

2. **a.** one less than the product of two and a number
b. two, times a number minus one
c. four plus the product of five and a number
d. the sum of four and five, times a number
e. six minus the product of three and a number
f. six minus three, times a number

3. **a.** $2n+3$ **b.** $4n-5$ **c.** $2(n+5)$ **d.** $4(n-1)$

4. **Step 1:** Write a variable for the number of nickels.

Let the number of nickels be n.

Step 2: Write a variable expression for the value in cents of the nickels.

Let the value of the nickels be $5n$.

Step 3: Find the value of the nickels if there are 10, 12, or 14 nickels.

- If $n=10$, then

$$\begin{aligned} &5n \\ &=5\times10 \\ &=50 \end{aligned}$$

If there are 10 nickels, the coins are worth 50¢.

- If $n=12$, then

$$\begin{aligned} &5n \\ &=5\times12 \\ &=60 \end{aligned}$$

If there are 12 nickels, the coins are worth 60¢.

- If $n=14$, then

$$\begin{aligned} &5n \\ &=5\times14 \\ &=70 \end{aligned}$$

If there are 14 nickels, the coins are worth 70¢.

5. **Step 1:** Write a variable for the number of posts.

Let the number of posts be n.

Step 2: Write a variable expression for the number of chains.

Let the number of chains be $n-1$.

Step 3: Find the number of chains if there are 2, 3, or 4 posts.

- If $n = 2$, then

$$\begin{aligned} & n-1 \\ &= 2-1 \\ &= 1 \end{aligned}$$

If there are 2 posts, there is 1 chain.

- If $n = 3$, then

$$\begin{aligned} & n-1 \\ &= 3-1 \\ &= 2 \end{aligned}$$

If there are 3 posts, there are 2 chains.

- If $n = 4$, then

$$\begin{aligned} & n-1 \\ &= 4-1 \\ &= 3 \end{aligned}$$

If there are 4 posts, there are 3 chains.

6. Step 1: Write a variable for the number of posts.

Let the number of posts be n.

Step 2: Write a variable expression for the number of crosspieces.

Let the number of crosspieces be $4n-4$.

Step 3: Find the number of crosspieces if there are 2, 3, or 4 posts.

- If $n = 2$, then

$$\begin{aligned} & 4n-4 \\ &= (4\times 2)-4 \\ &= 8-4 \\ &= 4 \end{aligned}$$

If there are 2 posts, there are 4 crosspieces.

- If $n = 3$, then

$$\begin{aligned} & 4n-4 \\ &= (4\times 3)-4 \\ &= 12-4 \\ &= 8 \end{aligned}$$

If there are 3 posts, there are 8 crosspieces.

- If $n = 4$, then

$$\begin{aligned} & 4n - 4 \\ &= (4 \times 4) - 4 \\ &= 16 - 4 \\ &= 12 \end{aligned}$$

If there are 4 posts, there are 12 crosspieces.

7. - If $n = 100$, then

$$\begin{aligned} & 12 + 0.05n \\ &= 12 + (0.05 \times 100) \\ &= 12 + 5 \\ &= 17 \end{aligned}$$

If there are 100 invitations, the cost of printing is \$17.

- If $n = 200$, then

$$\begin{aligned} & 12 + 0.05n \\ &= 12 + (0.05 \times 200) \\ &= 12 + 10 \\ &= 22 \end{aligned}$$

If there are 200 invitations, the cost of printing is \$22.

- If $n = 300$, then

$$\begin{aligned} & 12 + 0.05n \\ &= 12 + (0.05 \times 300) \\ &= 12 + 15 \\ &= 27 \end{aligned}$$

If there are 300 invitations, the cost of printing is \$27.

8. **a.**

a	$a+2$	Pattern
1	3	
2	4	+1
3	5	+1
4	6	+1
5	7	+1
6	8	+1

b.

c	$2c$	Pattern
1	2	
2	4	+2
3	6	+2
4	8	+2
5	10	+2
6	12	+2

c.

d	$3d+2$	Pattern
1	5	
2	8	+3
3	11	+3
4	14	+3
5	17	+3
6	20	+3

d.

b	$4b-1$	Pattern
1	3	
2	7	+4
3	11	+4
4	15	+4
5	19	+4
6	23	+4

9. In question 8.a. the pattern is add 1, and the variable expression is $a+2$ or $1a+2$.

In question 8.b. the pattern is add 2, and the variable expression is $2c$.

In question 8.c. the pattern is add 3, and the variable expression is $3d+2$.

In question 8.d. the pattern is add 4, and the variable expression is $4b-1$.

10. **a.**

k	$k-3$	Pattern
1	1	
2	5	+4
3	9	+4
4	13	+4

The pattern is add 4, so the variable expression is $4k-3$.

b.

y	$y+1$	Pattern
1	7	
2	13	+6
3	19	+6
4	25	+6

The pattern is add 6, so the variable expression is $6y+1$.

c.

d	$d-1$	Pattern
1	1	
2	3	+2
3	5	+2
4	7	+2

The pattern is add 2, so the variable expression is $2d-1$.

d.

x	$x+4$	Pattern
1	7	
2	10	+3
3	13	+3
4	16	+3

The pattern is add 3, so the variable expression is $3x+4$.

Section 1: Activity 3

1. **a.** The equal sign signals that the two expressions have the same value.

b. The equal sign signals that 17 is the value of the expression $5\times3+2$.

c. The equal sign signals two equivalent but different forms of a number; $\frac{5}{3}$ is the improper fraction form and $1\frac{2}{3}$ is the mixed number form.

2. **a.** $1+3=4$ **b.** $8-3=5$

c. $2\times(1+3)=8$ **d.** $2\times4+3=3\times5-4$

3. **a.** $12-n=4$ **b.** $6n=48$

c. $n-2=7$ **d.** $m=52-3$

e. $m\div2=78$ **f.** $2n-9=27$

4. **a.** The capital of Canada is Ottawa .

b. The boiling point of water is 100 °C.

c. 0 °C is the freezing point of water.

d. The hummingbird is the smallest bird.

5. **a.** $8+5=13$ **b.** $5+4=9$

c. $8-3=1+4$ **d.** $4+1=7-2$

6. **a.** $n+8=12$

$n=4$

LS	RS
$n+8$	12
$=4+8$	
$=12$	

LS = RS

b. $p-5=5$

$p=10$

LS	RS
$p-5$	5
$=10-5$	
$=5$	

LS = RS

c. $4b=12$

$b=3$

LS	RS
$4b$	12
$=4\times3$	
$=12$	

LS = RS

d. $6+x=13$

$x=7$

LS	RS
$6+x$	13
$=6+7$	
$=13$	

LS = RS

7. **a.** $3a+2=14$

$a=4$

LS	RS
$3a+2$	14
$=3\times4+2$	
$=12+2$	
$=14$	

LS = RS

b. $2(a-7)=6$

$a=10$

LS	RS
$2(a-7)$	6
$=2(10-7)$	
$=2\times3$	
$=6$	

LS = RS

c. $16=3s+1$

$s=5$

LS	RS
16	$3s+1$
	$=3\times5+1$
	$=15+1$
	$=16$

LS = RS

d. $3(a+4)=27$

$a=5$

LS	RS
$3(a+4)$	27
$=3(5+4)$	
$=3\times9$	
$=27$	

LS = RS

Now Try This

8. You can use the guess, check, and revise problem-solving strategy to solve this problem. There are many solutions. How many did you find? Here are some possible solutions.

- The number 5 is a solution.

$$5 = 1 + 4 \qquad 5^2 = 25$$
$$= 1^2 + 2^2 \qquad = 16 + 9$$
$$= 4^2 + 3^2$$

- The number 10 is a solution.

$$10 = 9 + 1 \qquad 10^2 = 100$$
$$= 3^2 + 1^2 \qquad = 36 + 64$$
$$= 6^2 + 8^2$$

- The number 13 is a solution.

$$13 = 4 + 9 \qquad 13^2 = 169$$
$$= 2^2 + 3^2 \qquad = 144 + 25$$
$$= 12^2 + 5^2$$

- The number 17 is a solution.

$$17 = 1 + 16 \qquad 17^2 = 289$$
$$= 1^2 + 4^2 \qquad = 225 + 64$$
$$= 15^2 + 8^2$$

- The number 25 is a solution.

$$25 = 9 + 16 \qquad 25^2 = 625$$
$$= 3^2 + 4^2 \qquad = 225 + 400$$
$$= 15^2 + 20^2$$

- The number 26 is a solution.

$$26 = 25 + 1 \qquad 26^2 = 676$$
$$= 5^2 + 1^2 \qquad = 100 + 576$$
$$= 10^2 + 24^2$$

- The number 50 is a solution.

$$50 = 25 + 25 \qquad 50^2 = 2500$$
$$= 5^2 + 5^2 \qquad = 1600 + 900$$
$$= 40^2 + 30^2$$

Section 1: Follow-up Activities

Extra Help

1. a.
$$6 + 4 \times 2$$
$$= 6 + 8$$
$$= 14$$

b.
$$14 + 8 \div 2^2$$
$$= 14 + 8 \div 2 \times 2$$
$$= 14 + 8 \div 4$$
$$= 14 + 2$$
$$= 16$$

c.
$$3 \times (9 + 4)$$
$$= 3 \times 13$$
$$= 39$$

d. $(6-5)\times 3$
$= 1\times 3$
$= 3$

e. $4\times 5+7$
$= 20+7$
$= 27$

f. $\left(7+3^2\right)\div 2$
$= (7+3\times 3)\div 2$
$= (7+9)\div 2$
$= 16\div 2$
$= 8$

g. $2+8\times 4\div 2$
$= 2+32\div 2$
$= 2+16$
$= 18$

h. $10-8\div 4\times 3$
$= 10-2\times 3$
$= 10-6$
$= 4$

i. $3^2+3\times 4-1$
$= 3\times 3+3\times 4-1$
$= 9+12-1$
$= 21-1$
$= 20$

2. a.

If $m=1$, then	If $m=2$, then	If $m=3$, then
$5m$	$5m$	$5m$
$=5\times 1$	$=5\times 2$	$=5\times 3$
$=5$	$=10$	$=15$

b.

If $s=3$, then	If $s=4$, then	If $s=5$, then
$s+2$	$s+2$	$s+2$
$=3+2$	$=4+2$	$=5+2$
$=5$	$=6$	$=7$

c.

If $x=2$, then	If $x=4$, then	If $x=6$, then
$2x-3$	$2x-3$	$2x-3$
$=2\times 2-3$	$=2\times 4-3$	$=2\times 6-3$
$=4-3$	$=8-3$	$=12-3$
$=1$	$=5$	$=9$

d.

If $m=1$, then	If $m=2$, then	If $m=3$, then
$3m+1$	$3m+1$	$3m+1$
$=3\times 1+1$	$=3\times 2+1$	$=3\times 3+1$
$=3+1$	$=6+1$	$=9+1$
$=4$	$=7$	$=10$

3. a. $12+a=15$
$a=3$

LS	RS
$12+a$	15
$=12+3$	
$=15$	

LS = RS

b. $2n=18$
$n=9$

LS	RS
$2n$	18
$=2\times 9$	
$=18$	

LS = RS

c. $b-5=3$

$b=8$

LS	RS
$b-5$	3
$=8-5$	
$=3$	

LS = RS

d. $3a+2=14$

$a=4$

LS	RS
$3a+2$	14
$=3\times4+2$	
$=12+2$	
$=14$	

LS = RS

e. $2x-1=9$

$x=5$

LS	RS
$2x-1$	9
$=2\times5-1$	
$=10-1$	
$=9$	

LS = RS

f. $2(a+5)=14$

$a=2$

LS	RS
$2(a+5)$	14
$=2(2+5)$	
$=2\times7$	
$=14$	

LS = RS

Enrichment

1. a. $10-[(4+8)\div3]$

$= 10-[12\div3]$

$= 10-4$

$= 6$

b. $10\times[6\div(8-5)]$

$= 10\times[6\div3]$

$= 10\times2$

$= 20$

c. $[(3+2)\times(5-3)]\div2$

$= [5\times2]\div2$

$= 10\div2$

$= 5$

d. $4+6\times[2\times(6-3)]$

$= 4+6\times[2\times3]$

$= 4+6\times6$

$= 4+36$

$= 40$

2. a. If $x=2.5$, then $5x+2=14.5$.

B4 | × | √ | = 5 * B3 + 2

	A	B
1	Evaluate 5x + 2	
2		
3	x	2.5
4	5x + 2	14.5
5		

If $x=3.8$, then $5x+2=21$.

B4 | × | √ | = 5 * B3 + 2

	A	B
1	Evaluate 5x + 2	
2		
3	x	3.8
4	5x + 2	21
5		

If $x = 5.7$, then $5x + 2 = 30.5$.

B4 | × | √ | = 5 * B3 + 2

	A	B
1	Evaluate 5x + 2	
2		
3	x	5.7
4	5x + 2	30.5
5		

b. If $s = 1.9$, then $7s - 1 = 12.3$.

B4 | × | √ | = 7 * B3 - 1

	A	B
1	Evaluate 7s - 1	
2		
3	s	1.9
4	7s - 1	12.3
5		

If $s = 2.4$, then $7s - 1 = 15.8$.

B4 | × | √ | = 7 * B3 - 1

	A	B
1	Evaluate 7s - 1	
2		
3	s	2.4
4	7s - 1	15.8
5		

If $s = 4.2$, then $7s - 1 = 28.4$.

B4 | × | √ | = 7 * B3 - 1

	A	B
1	Evaluate 7s - 1	
2		
3	s	4.2
4	7s - 1	28.4
5		

c. If $n = 10$, then $8.2n + 3.2 = 85.2$.

B4 | × | √ | = 8.2 * B3 + 3.2

	A	B
1	Evaluate 8.2n + 3.2	
2		
3	n	10
4	8.2n + 3.2	85.2
5		

If $n = 12$, then $8.2n + 3.2 = 101.6$.

B4 | × | √ | = 8.2 * B3 + 3.2

	A	B
1	Evaluate 8.2n + 3.2	
2		
3	n	12
4	8.2n + 3.2	101.6
5		

If $n = 20$, then $8.2n + 3.2 = 167.2$.

B4 | ✗ ✓ | = 8.2 * B3 + 3.2

	A	B
1	Evaluate 8.2n + 3.2	
2		
3	n	20
4	8.2n + 3.2	167.2
5		

Section 2: Activity 1

1. a. $\{(\text{Manuel, M}), (\text{Eva, E}), (\text{Peter, P}), (\text{Kyoko, K}), (\text{Nina, N})\}$

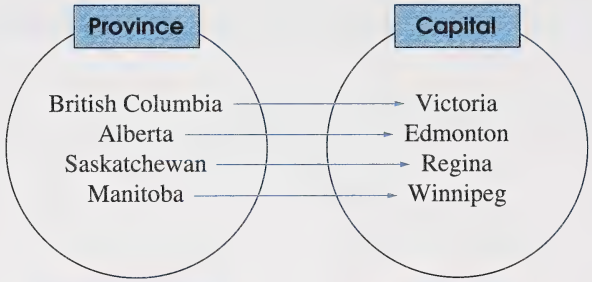
b. $\{(\text{basketball, Manuel}), (\text{hockey, Eva}), (\text{figure skating, Peter}), (\text{softball, Kyoko}), (\text{football, Nina})\}$

2. Answers may vary.

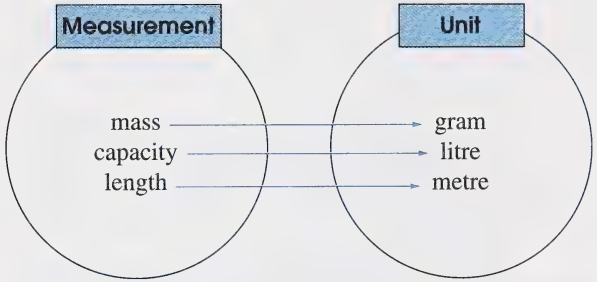
a. The relation rule for question 1.a. is "begins with the letter."

b. The relation rule for question 1.b. is "is the favourite sport of."

3. a.



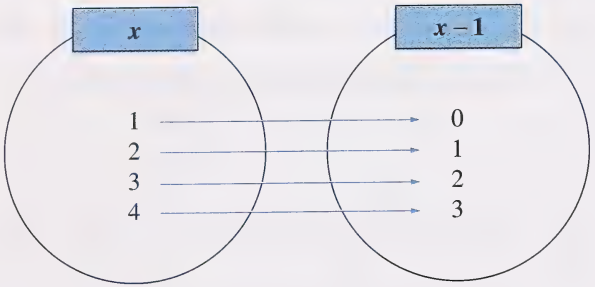
b.



4. a. $\{(\text{Jacques, Marie}), (\text{Franz, Joanne}), (\text{Jeanne, Yvonne}), (\text{Mary, Ruth}), (\text{Ichiro, Isabelle})\}$

b. $\{(\text{centimetres, cm}), (\text{degrees Celsius, °C}), (\text{kilograms, kg}), (\text{millilitres, mL})\}$

5. a.



If $x = 1$, then

$x - 1$
$= 1 - 1$
$= 0$

If $x = 2$, then

$x - 1$
$= 2 - 1$
$= 1$

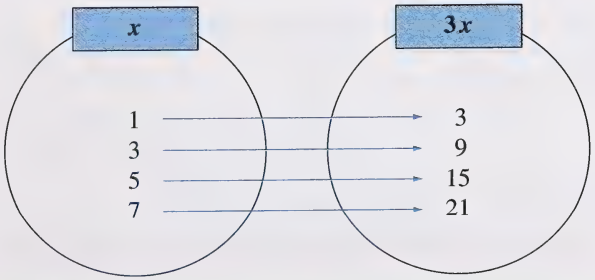
If $x = 3$, then

$x - 1$
$= 3 - 1$
$= 2$

If $x = 4$, then

$x - 1$
$= 4 - 1$
$= 3$

b.



If $x = 1$, then

$3x$
$= 1 \times 3$
$= 3$

If $x = 3$, then

$3x$
$= 3 \times 3$
$= 9$

If $x = 5$, then

$3x$
$= 3 \times 5$
$= 15$

If $x = 7$, then

$3x$
$= 3 \times 7$
$= 21$

6. a.

Shape	Number of Squares	Number of Dots	Pattern
	2	1	← 2 − 1
	3	2	← 3 − 1
	4	3	← 4 − 1

b.

Shape	Number of Squares	Number of Dots	Pattern
	2	1	← 2 – 1
	3	2	← 3 – 1
	4	3	← 4 – 1
	5	4	← 5 – 1
	6	5	← 6 – 1
	7	6	← 7 – 1

c.

If there are five squares, then there are 4 dots.

If there are six squares, then there are 5 dots.

If there are seven squares, then there are 6 dots.

d. If there are n squares, then there are $n-1$ dots.

e. The relation rule may be written as $n \longrightarrow n-1$.

f. If $n = 25$, then

$$\begin{aligned} & n-1 \\ &= 25-1 \\ &= 24 \end{aligned}$$

If there are 25 squares, there will be 24 dots.

7. a.

Shape	Total Number of Toothpicks	Number of Vertical Toothpicks	Pattern
	3	1	← 3 ÷ 3
	6	2	← 6 ÷ 3
	9	3	← 9 ÷ 3

b.

Shape	Total Number of Toothpicks	Number of Vertical Toothpicks	Pattern
	3	1	← $3 \div 3$
	6	2	← $6 \div 3$
	9	3	← $9 \div 3$
	12	4	← $12 \div 3$
	15	5	← $15 \div 3$
	18	6	← $18 \div 3$

c.

If there are 12 toothpicks, then there are 4 vertical toothpicks.

If there are 15 toothpicks, then there are 5 vertical toothpicks.

If there are 18 toothpicks, then there are 6 vertical toothpicks.

d. If there are x toothpicks, then there are $x \div 3$ vertical toothpicks.

e. The relation rule may be written as $x \longrightarrow x \div 3$.

f. If $x = 30$, then

$$x \div 3$$
$$= 30 \div 3$$
$$= 10$$

If there are 30 toothpicks in the shape, there will be 10 vertical toothpicks.

8. a.

Shape	Number of Triangles	Number of Toothpicks	Pattern
	1	3	← 2×1+1
	2	5	← 2×2+1
	3	7	← 2×3+1

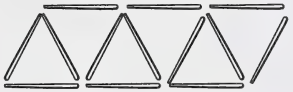
b.

Shape	Number of Triangles	Number of Toothpicks	Pattern
	1	3	← 2×1+1
	2	5	← 2×2+1
	3	7	← 2×3+1
	4	9	← 2×4+1
	5	11	← 2×5+1
	6	13	← 2×6+1

c.

If there are 4 triangles, then there are 9 toothpicks.

If there are 5 triangles, then there are 11 toothpicks.



If there are 6 triangles, then there are 13 toothpicks.

d. If there are n triangles, then there are $2n+1$ toothpicks.

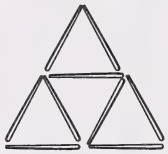
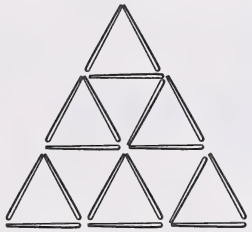
e. The relation rule may be written as $n \longrightarrow 2n+1$.

f. If $n = 27$, then

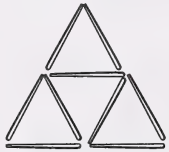
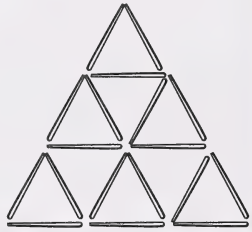
$$\begin{aligned} & 2n+1 \\ &= 2\times 27+1 \\ &= 54+1 \\ &= 55 \end{aligned}$$

If there are 27 triangles, then 55 toothpicks will be needed.

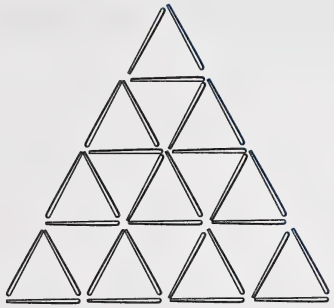
9. a.

Shape	Number of Toothpicks on the Base	Number of Small Triangles	Pattern
	1	1	$\leftarrow 1^2$
	2	4	$\leftarrow 2^2$
	3	9	$\leftarrow 3^2$

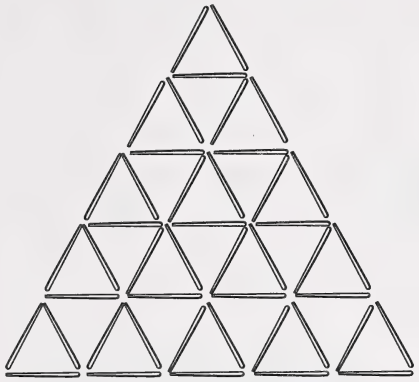
b.

Shape	Number of Toothpicks on the Base	Number of Small Triangles	Pattern
	1	1	$\leftarrow 1^2$
	2	4	$\leftarrow 2^2$
	3	9	$\leftarrow 3^2$
	4	16	$\leftarrow 4^2$
	5	25	$\leftarrow 5^2$
	6	36	$\leftarrow 6^2$

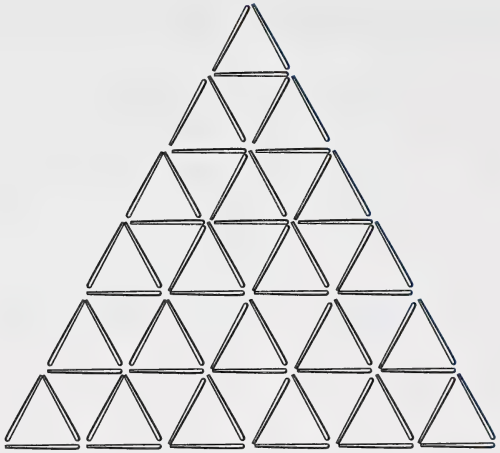
c.



If there are 4 toothpicks on the base, then there are 16 triangles.



If there are 5 toothpicks on the base, then there are 25 triangles.



If there are 6 toothpicks on the base, then there are 36 triangles.

d. If there are x toothpicks on the base, there are x^2 triangles.

e. The relation rule may be written as $x \longrightarrow x^2$.

f. If $x = 14$, then

$$x^2$$
$$= 14 \times 14$$
$$= 196$$

If there are 14 toothpicks on the base, then 196 triangles can be created.

10. a.

Number of Hours	Pay ($)	Pattern
1	15.25	← 15.25×1
2	30.50	← 15.25×2
3	45.75	← 15.25×3
4	61.00	← 15.25×4
...	...	
5	76.25	← 15.25×5

If Ruth works for 5 h, she will earn $76.25.

b. If Ruth works x hours, she will earn $15.25x$ dollars.

c. The relation rule may be written as $x \longrightarrow 15.25x$.

d. If $x = 40$, then

$$\begin{aligned} & 15.25x \\ &= 15.25 \times 40 \\ &= 610 \end{aligned}$$

So, Ruth earns $610 in an average work week.

11. a.

Riding Time (h)	Cost ($)	Pattern
1	6	← 2×1+4
2	8	← 2×2+4
3	10	← 2×3+4
4	12	← 2×4+4
5	14	← 2×5+4
...	...	
6	16	← 2×6+4

If Sylvia rides for 6 h, the cost will be $16.

b. If Sylvia rides for n hours, the cost will be $2n+4$ dollars.

c. The relation rule may be written as $n \longrightarrow 2n+4$.

d. **Step 1:** Find the total number of hours.

$$1.5+3+6=10.5$$

Step 2: Find the total cost.

If $n = 10.5$, then

$$\begin{aligned} & 2n+4 \\ &= 2 \times 10.5 + 4 \\ &= 21+4 \\ &= 25 \end{aligned}$$

If, Sylvia rides for 1.5 h on Monday, 3 h on Tuesday, and 6 h on Thursday, the cost will be $25.

12. a. 1932, 1944, 1956, 1968, 1980, 1992, 2004, 2016, 2028

+12 +12 +12 +12 +12 +12 +12 +12

The next three years of the Monkey will occur in 2004, 2016, and 2028.

b. …, 1896, 1908, 1920, 1932, 1944, 1956, 1968, 1980, 1992,…

The first year of the twentieth century was 1901. The first year of the Monkey in the twentieth century was 1908.

13. a. 3, 6, 9, 12, 15, 18, 21, 24, …, $3n$

3×1 3×2 3×3 3×4 3×5 3×6 3×7 3×8 $3n$

b. 1, 3, 5, 7, 9,

$2\times1-1$ $2\times2-1$ $2\times3-1$ $2\times4-1$ $2\times5-1$

11, 13, 15, …, $2n-1$

$2\times6-1$ $2\times7-1$ $2\times8-1$ $2n-1$

c. 4, 7, 10, 13, 16,

$3\times1+1$ $3\times2+1$ $3\times3+1$ $3\times4+1$ $3\times5+1$

19, 22, 25, …, $3n+1$

$3\times6+1$ $3\times7+1$ $3\times8+1$ $3n+1$

d. 1, 4, 9, 16, 25, 36, 49, 64, …, n^2

1^2 2^2 3^2 4^2 5^2 6^2 7^2 8^2 n^2

e. 2, 5, 10, 17, 26,

1^2+1 2^2+1 3^2+1 4^2+1 5^2+1

37, 50, 65, …, n^2+1

6^2+1 7^2+1 8^2+1 n^2+1

Now Try This

14. You may try working backwards. **Note:** The opposite of multiplying by 2 is dividing by 2.

On the 60th day the lake is completely covered.

$1 \div 2 = 0.5$

On the 59th day the lake is 0.5 covered.

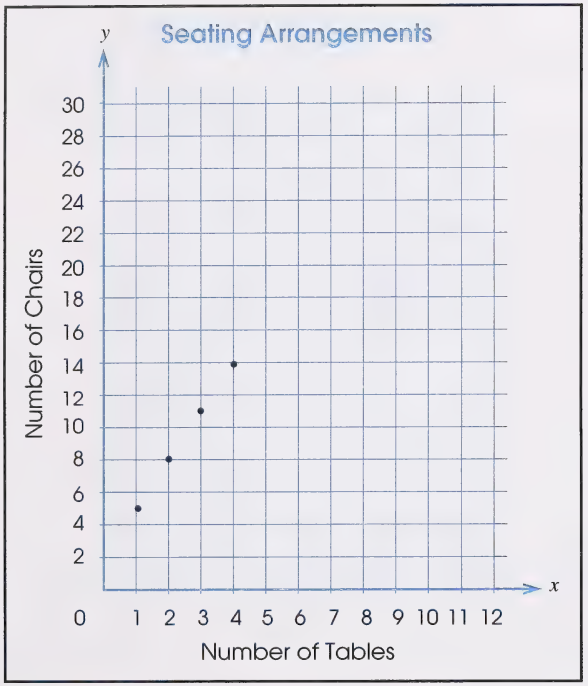
$0.5 \div 2 = 0.25$

On the 58th day the lake is 0.25 covered.

So, it takes 58 days for the lake to be 0.25 covered.

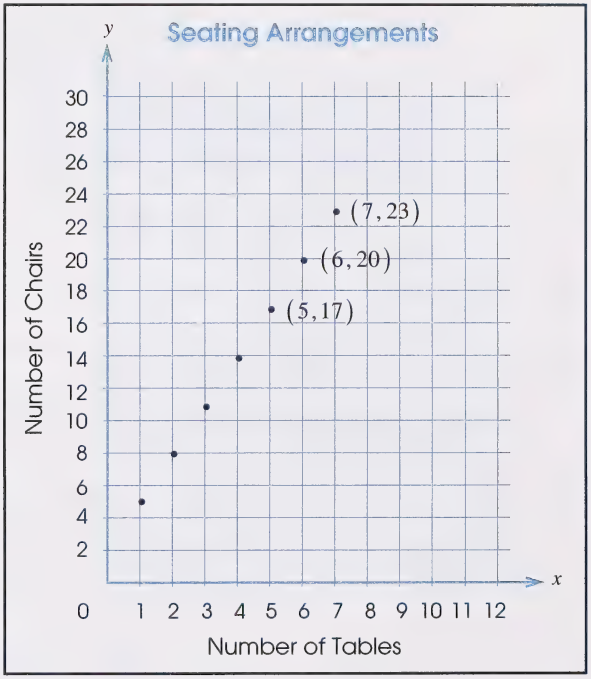
Section 2: Activity 2

1. a. The scales may vary on the x-axis or the y-axis.



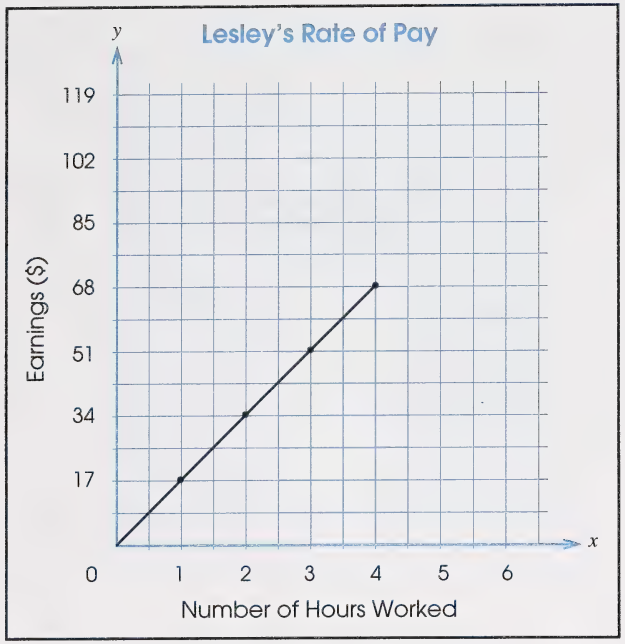
The dots are not connected because you cannot have a fraction of a table or a chair.

b. Extend the dots and predict the number of chairs for 5, 6, or 7 tables. (You are extrapolating.)



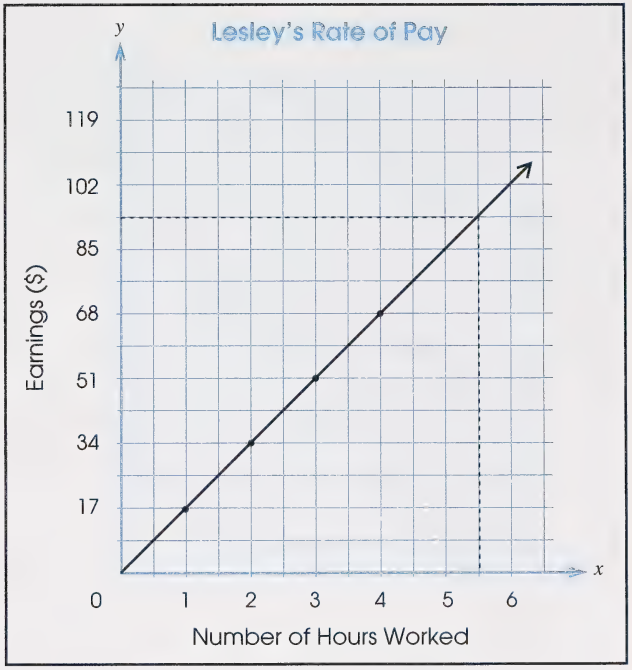
With 5 tables, there will be 17 chairs.
With 6 tables, there will be 20 chairs.
With 7 tables, there will be 23 chairs.

2. a. The scales may vary on the x-axis or the y-axis.



The dots are connected because you can have a fraction of a dollar.

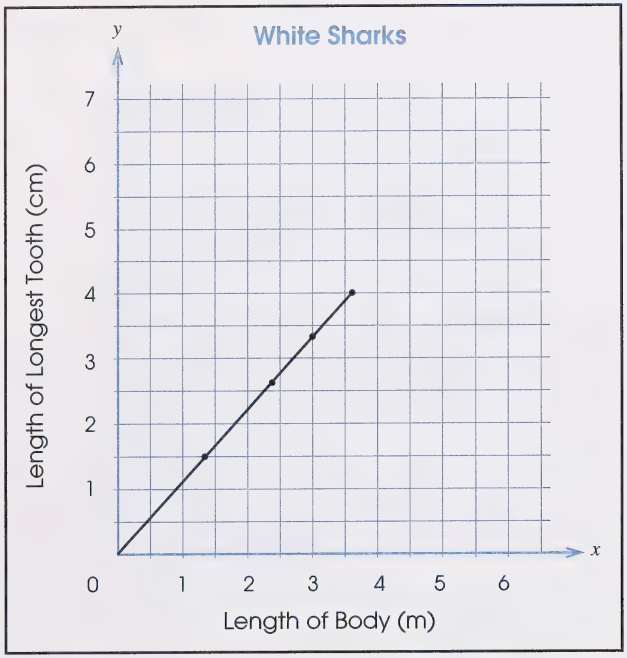
b. Extend the line and predict Lesley's earnings for 5.5 h. (You are extrapolating.)



Lesley will earn $93.50 when she works 5.5 h.

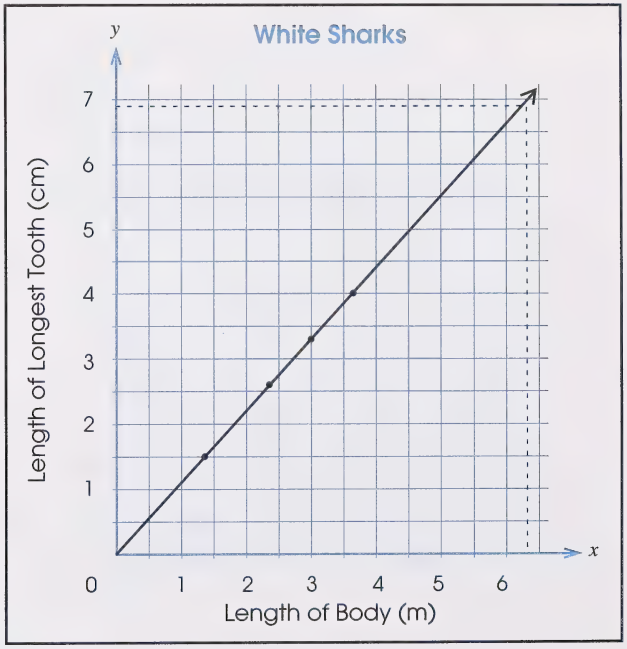
The arrowhead indicates the pattern continues.

3. a. The scales may vary on the x-axis or the y-axis.



The dots are connected because you can have a fraction of a centimetre or a metre.

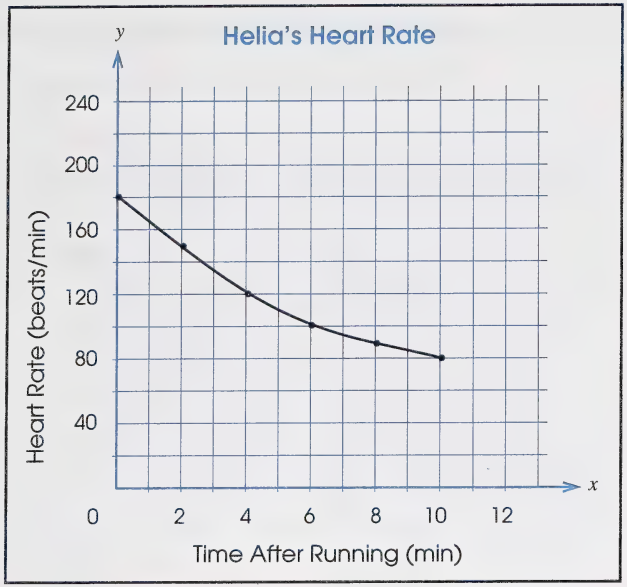
b. Extend the line and predict the length of the longest tooth of a 6.4-m shark. (You are extrapolating.)



The shark's longest tooth was probably about 6.9 cm.

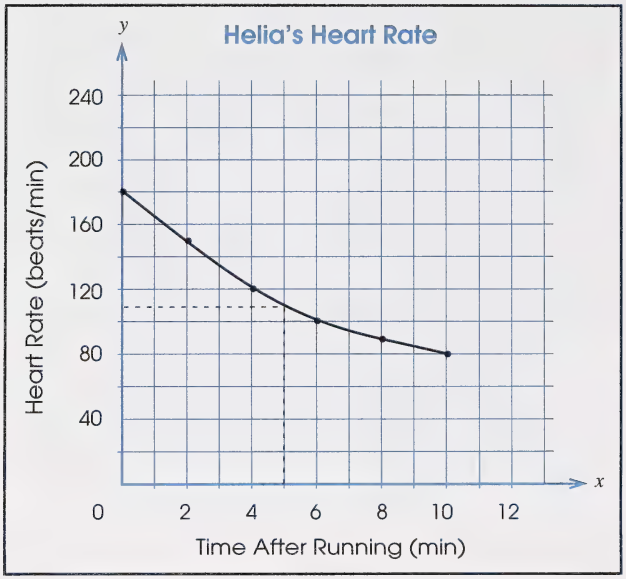
The arrowhead indicates the pattern continues; however, there is a limit to the size that sharks can grow so the pattern does not continue forever.

4. a.



The dots are connected because you can have a fraction of a minute.

b. Use the graph to calculate Helia's heart rate after running for 5 min. (You are interpolating.)



Helia's heart rate 5 min after running was about 110 beats/min.

5. a. Use the graph to list the ordered pairs and find a rule to describe the pattern.

Week Number	Savings ($)	Pattern
1	251	← 250 + 1
2	252	← 250 + 2
3	253	← 250 + 3
4	254	← 250 + 4
5	255	← 250 + 5
...	...	
n	$250 + n$	← $250 + n$

So, the relation rule is $n \longrightarrow 250 + n$.

b. The relation rule is $n \longrightarrow 250 + n$.

If $n = 52$, then

$$\begin{aligned} & 250 + n \\ &= 250 + 52 \\ &= 302 \end{aligned}$$

So, by the end of the year, Herbert could save $302.

Now Try This

6. You can solve the problem by finding and applying a pattern.

a. $0.5 \times 40 = 20$

The ball will reach 20 m after the first bounce.

b. $0.5 \times 20 = 10$

The ball will reach 10 m after the second bounce.

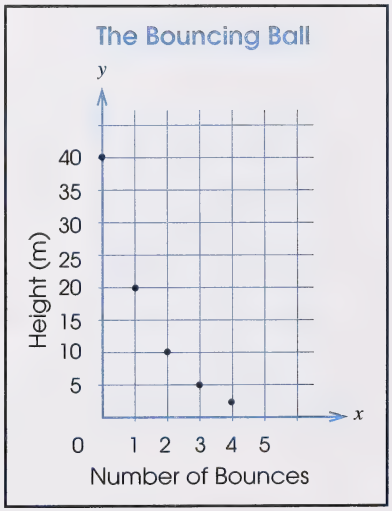
c. $0.5 \times 10 = 5$

The ball will reach 5 m after the third bounce.

d. $0.5 \times 5 = 2.5$

The ball will reach 2.5 m after the fourth bounce.

Note: A graph may help you visualize the pattern of the bounces.



Section 2: Follow-up Activities

Extra Help

1.

Point	Coordinates
A	$(5,9)$
B	$(3,8)$
C	$(7,8)$
D	$(2,6)$
E	$(8,6)$
F	$(1,4)$
G	$(9,4)$
H	$(0,2)$
I	$(10,2)$
J	$(5,0)$

2.

Point	Coordinates
A	$(1,9\frac{1}{2})$
B	$(2\frac{1}{2},11\frac{1}{2})$
C	$(4,12\frac{1}{2})$
D	$(6,13\frac{1}{2})$
E	$(7,13\frac{1}{2})$
F	$(9,12\frac{1}{2})$
G	$(10,11\frac{1}{2})$
H	$(12,9\frac{1}{2})$
I	$(5,3\frac{1}{2})$
J	$(3\frac{1}{2},5\frac{1}{2})$

Enrichment

1. a. $x \longrightarrow y$
$x \longrightarrow x+2$

So, $y = x+2$.

b. $x \longrightarrow y$
$x \longrightarrow 3x$

So, $y = 3x$.

c. $x \longrightarrow y$
$x \longrightarrow 2x+3$

So, $y = 2x+3$.

2. a. If $x = 10$, then

$x+2$
$= 10+2$
$= 12$

So, $y = 12$.

b. If $x = 10$, then

$3x$
$= 3\times 10$
$= 30$

So, $y = 30$.

c. If $x = 10$, then

$2x+3$
$= 2\times 10+3$
$= 20+3$
$= 23$

So, $y = 23$.

Section 3: Activity 1

1. Because of the measurement methods, answers may vary slightly.

- The perimeter of Rectangle A is 21 cm.
- The perimeter of Rectangle B is 24 cm.
- The perimeter of Rectangle C is 12 cm.
- The perimeter of Rectangle D is 15 cm.
- The perimeter of Rectangle E is 26 cm.

2. The perimeter is equal to twice the length plus twice the width.

3. $P = 2\ell + 2w$
$= (2\times 6)+(2\times 4.5)$
$= 12+9$
$= 21$

Usually the longer side of a rectangle is called the length.

The perimeter of the room is 21 m.

4. $P = 2\ell + 2w$

$= (2 \times 23.8) + (2 \times 11)$

$= 47.6 + 22$

$= 69.6$

The perimeter of the tennis court is 69.6 m.

5. **a.** The area is 18 square units.
b. The area is 20 square units.
c. The area is 16 square units.
d. The area is 21 square units.

6. The area is equal to the length times the width.

7. $A = \ell w$

$= 15.2 \times 7$

$= 106.4$

The area of a Canadian $10 bill is 106.4 cm^2.

8. $A = \ell w$

$= 3 \times 1.5$

$= 4.5$

The area of a Canadian flag is 4.5 m^2.

9. **a.**

Rectangles with an Area of 16 Square Units		
Length (units)	Width (units)	Perimeter (units)
1	16	34
2	8	20
4	4	16
8	2	20
16	1	34

b. The rectangle with a length of 1 unit and the rectangle with a length of 16 units have the greatest perimeter.

c. The rectangle with a length of 4 units has the least perimeter.

10. **a.**

Rectangles with a Perimeter of 24 Units		
Length (units)	Width (units)	Area (square units)
1	11	11
2	10	20
3	9	27
4	8	32
5	7	35
6	6	36
7	5	35
8	4	32
9	3	27
10	2	20
11	1	11

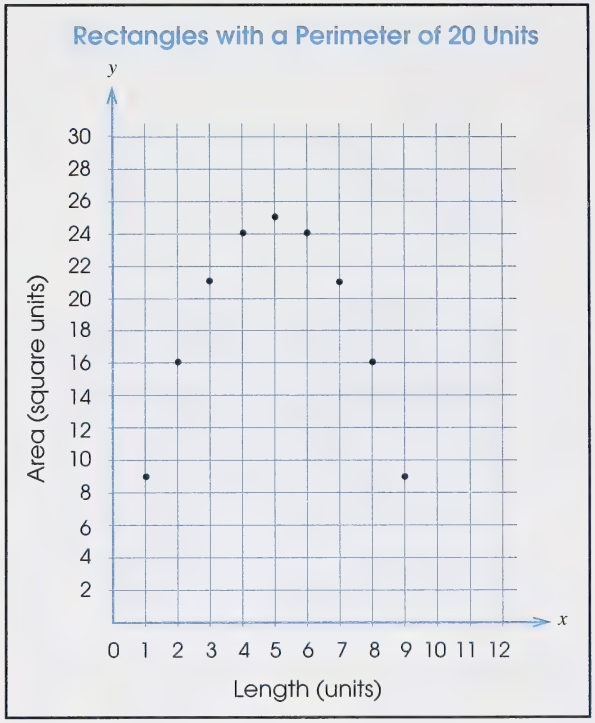
b. The rectangle with the greatest area has a length of 6 units.

c. The rectangle with a length of 1 unit and the rectangle with a length of 11 units have the least area.

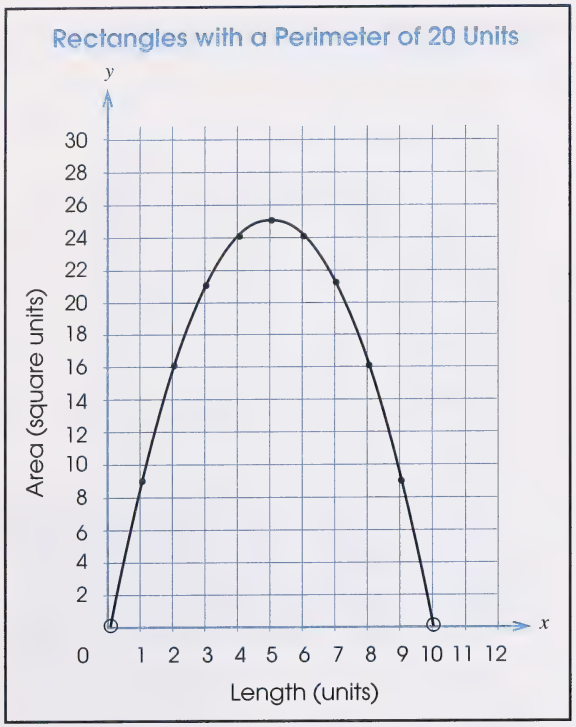
11. a.

Rectangles with a Perimeter of 20 Units		
Length (units)	**Width (units)**	**Area (square units)**
1	9	9
2	8	16
3	7	21
4	6	24
5	5	25
6	4	24
7	3	21
8	2	16
9	1	9

b. The scales may vary on the x-axis and the y-axis.



c.



The dots are joined because it is possible to have a length with a fraction of a unit.

The open dots on the x-axis indicate there is no rectangle with a length of 0 units or 10 units.

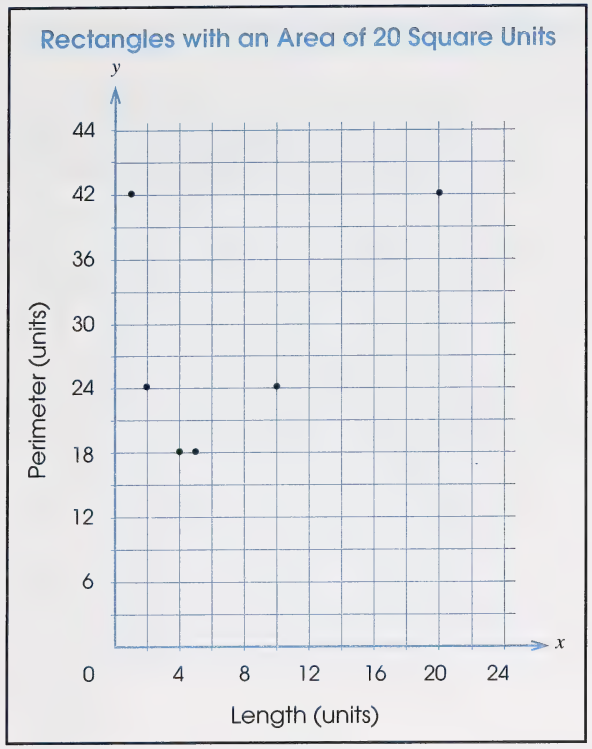
d. From the graph you can see that as the length increases, the area increases. The maximum area is for the rectangle with a length of 5 units; this is the turning point. After that, as the length increases, the area decreases.

e. The rectangle that is 5 units by 5 units has the greatest area. This rectangle is a square.

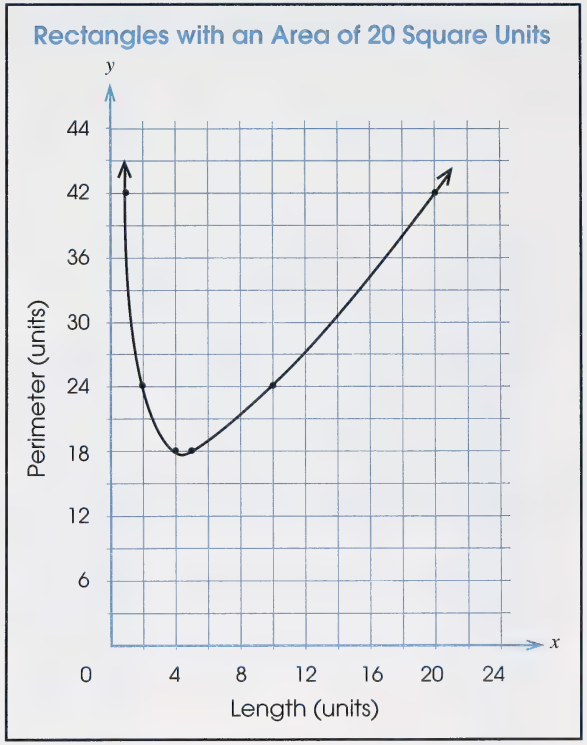
12. a.

Rectangles with an Area of 20 Square Units		
Length (units)	**Width (units)**	**Perimeter (units)**
1	20	42
2	10	24
4	5	18
5	4	18
10	2	24
20	1	42

b.



c.

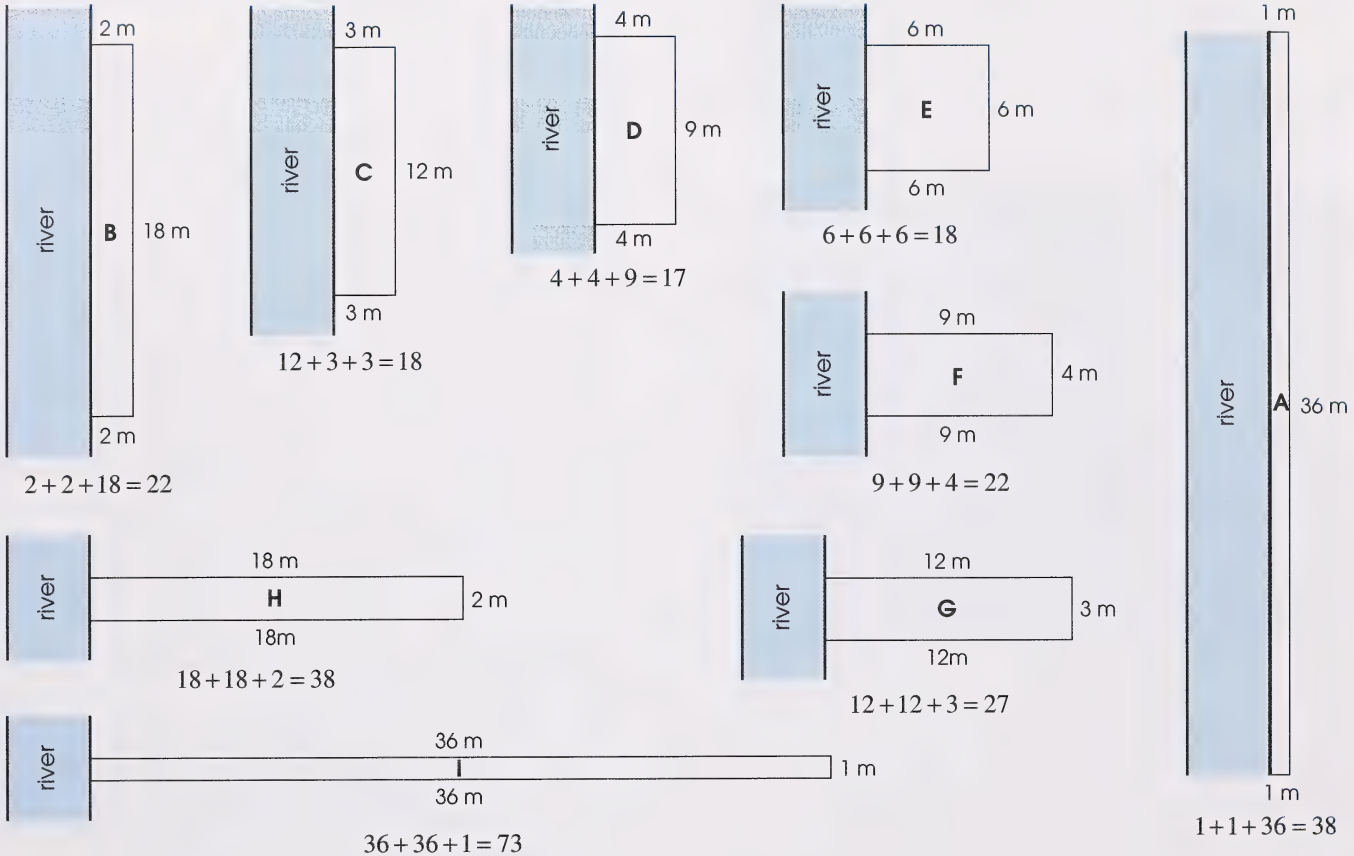


The dots are joined because it is possible to have a length that is a fraction of a unit.

d. As the lengths of the rectangles increase, the perimeters decrease. The least perimeter is for a rectangle with a length of about 4.5 units; this is the turning point. After that, as the lengths increase, the perimeters increase.

Now Try This

13. a.



b. The most economical design is like Pen D.

c. The most expensive design is like Pen I.

14. You may list the rectangles with an area of 144 m^2 to find the one with the least perimeter.

Rectangles with an Area of 144 m^2		
Length (m)	**Width (m)**	**Perimeter (m)**
1	144	290
2	72	148
3	48	102
4	36	80
6	24	60
8	18	52
9	16	50
12	12	48 ← minimum perimeter
16	9	50
18	8	52
24	6	60
36	4	80
48	3	102
72	2	148
144	1	290

The rectangle with a length of 12 m and a width of 12 m has the least perimeter.

So, the dog run should be 12 m by 12 m.

15. You can use the guess, check, and revise strategy to solve the problem. The width is 10 m. The length is 11 m.

16. You can use elimination to solve the problem.

Step 1: Consider the clues. Who is tallest?

The clues indicate the following:

- Mario and Sharon are not the tallest. They are shorter than Kyla.
- Tom is not the tallest. Sharon is taller.
- Barb is not the tallest. Barb is the second tallest.

The clues can be shown on a table.

Height Order	Kyla	Tom	Barb	Sharon	Mario
Tallest		✗	✗	✗	✗
Second tallest					
Third tallest					
Fourth tallest					
Shortest					

It is clear that Kyla is the tallest.

Eliminate Kyla as the second tallest, third tallest, fourth tallest, and shortest.

Height Order	Kyla	Tom	Barb	Sharon	Mario
Tallest	✔	✗	✗	✗	✗
Second tallest	✗				
Third tallest	✗				
Fourth tallest	✗				
Shortest	✗				

Step 2: Reconsider the clues. Who is the second tallest?

The clues indicate that Barb is the second tallest. (Eliminate Barb as the third tallest, fourth tallest and shortest.)

Height Order	Kyla	Tom	Barb	Sharon	Mario
Tallest	✔	✗	✗	✗	✗
Second tallest	✗	✗	✔	✗	✗
Third tallest	✗		✗		
Fourth tallest	✗		✗		
Shortest	✗		✗		

Step 3: Reconsider the clues. Who is third tallest?

The clues indicate the following:

- Sharon is not the third tallest. She is shorter than Mario.
- Tom is not the third tallest. Sharon is taller.

Height Order	Kyla	Tom	Barb	Sharon	Mario
Tallest	✔	✗	✗	✗	✗
Second tallest	✗	✗	✔	✗	✗
Third tallest	✗	✗	✗	✗	
Fourth tallest	✗		✗		
Shortest	✗		✗		

By elimination it is clear that Mario is the third tallest. (Eliminate Mario as the fourth tallest and shortest.)

Height Order	Kyla	Tom	Barb	Sharon	Mario
Tallest	✔	✗	✗	✗	✗
Second tallest	✗	✗	✔	✗	✗
Third tallest	✗	✗	✗	✗	✔
Fourth tallest	✗		✗		✗
Shortest	✗		✗		✗

Step 4: Reconsider the clues. Who is fourth tallest and who is shortest?

Tom is not fourth tallest. Sharon is taller.

Height Order	Kyla	Tom	Barb	Sharon	Mario
Tallest	✔	✗	✗	✗	✗
Second tallest	✗	✗	✔	✗	✗
Third tallest	✗	✗	✗	✗	✔
Fourth tallest	✗	✗	✗		✗
Shortest	✗		✗		✗

By elimination it is clear that Sharon is fourth tallest and Tom is shortest.

Height Order	Kyla	Tom	Barb	Sharon	Mario
Tallest	✔	✗	✗	✗	✗
Second tallest	✗	✗	✔	✗	✗
Third tallest	✗	✗	✗	✗	✔
Fourth tallest	✗	✗	✗	✔	✗
Shortest	✗	✔	✗	✗	✗

So, the students in order of height are Kyla, Barb, Mario, Sharon, and Tom.

Section 3: Activity 2

1. **a.** The volume is 5 cubic units.
b. The volume is 60 cubic units.
c. The volume is 24 cubic units.
d. The volume is 12 cubic units.

2. The volume is equal to the length times the width times the height.

3. $V = \ell wh$
$= 2 \times 15 \times 7$
$= 30 \times 7$
$= 210$ The volume is 210 cm^3.

4. $V = \ell wh$
$= 2.2 \times 2.4 \times 5.5$
$= 29.04$
$\doteq 29.0$ The volume is about 29.0 m^3.

Now Try This

5. To solve this problem you can use objects such as 27 blocks or sugar cubes. Stack the blocks or cubes as shown in the diagram. Then mark the parts that would be painted. (You could put a sticker or a ✔ on the faces that would be painted.)

a. There will be 8 cubes with three faces painted.

b. There will be 12 cubes with two faces painted.
c. There will be 6 cubes with one face painted.
d. There will be 1 cube with no paint at all.

Section 3: Activity 3

1. Because of the measurement methods, answers may vary slightly.

Circle A has a circumference of about 18.8 cm.
Circle B has a circumference of about 12.6 cm.
Circle C has a circumference of about 15.7 cm.
Circle D has a circumference of about 9.4 cm.
Circle E has a circumference of about 22 cm.

2. a. Because of the measurement methods, answers will vary slightly.

Circle	Circumference (cm)	Diameter (cm)	Ratio of the Circumference to the Diameter
Circle A	18.8	6	3.13
Circle B	12.6	4	3.15
Circle C	15.7	5	3.14
Circle D	9.4	3	3.13
Circle E	22	7	3.14

b. The ratio of the circumference to the diameter for each circle is between 3 and 4. If you could measure exactly, the ratio of the circumference to the diameter for each circle, rounded to the nearest hundredth, would be 3.14.

3. The ratio of the circumference to the diameter for each circle is about 3.14. This value is equal to π. So, the circumference of a circle is equal to pi times the diameter.

4. $C = \pi d$
$\doteq 3.14 \times 4.27$
$\doteq 13.41$

The golf ball has a circumference of about 13.41 cm.

5. $C = \pi d$
$\doteq 3.14 \times 3.66$
$\doteq 11.49$

The circumference of the largest ring is about 11.49 m.

$C = \pi d$
$\doteq 3.14 \times 2.44$
$\doteq 7.66$

The circumference of the middle ring is about 7.66 m.

$C = \pi d$
$\doteq 3.14 \times 1.22$
$\doteq 3.83$

The circumference of the smallest ring is about 3.83 m.

Now Try This

6. Use Venn diagrams to solve the problem.

Step 1: Perform the operations.

Find the number of women who wore a necklace.

$$20 \times 0.75 = 15$$

So, 15 women wore a necklace.

Find the number of women who wore a watch.

$$20 \times 0.8 = 16$$

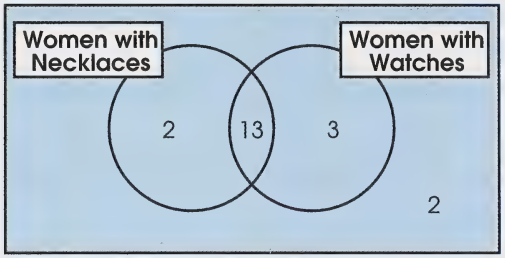
So, 16 women wore a watch.

Find the number of women who wore neither a watch nor a necklace.

$$20 \times 0.1 = 2$$

So, 2 women wore neither a watch nor a necklace.

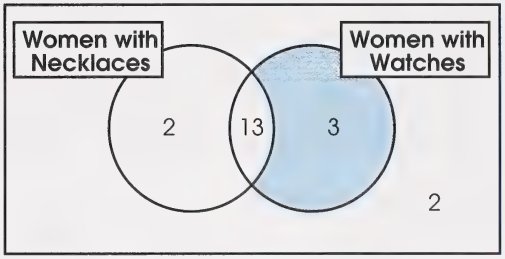
Step 2: Use the guess, check, and revise strategy with a Venn diagram to organize the numbers so that they total 20.



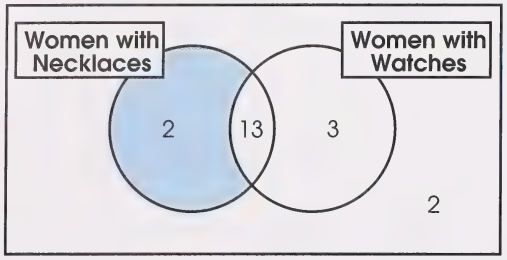
$$2 + 13 + 3 + 2 = 20$$

Step 3: Use the Venn diagram to answer the questions.

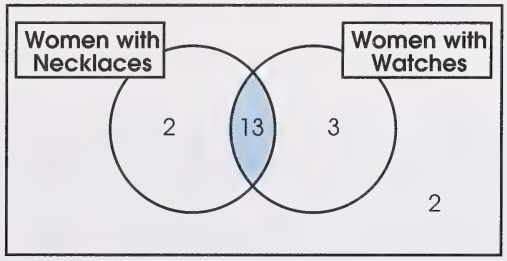
a. Of the 20 women, 3 wore only a watch for jewellery.



b. Of the 20 women, 2 wore only a necklace for jewellery.



c. Of the 20 women, 13 wore both a watch and a necklace for jewellery.



Section 3: Follow-up Activities

Extra Help

1. a. $P = 2\ell + 2w$
$= 2 \times 7 + 2 \times 3$
$= 14 + 6$
$= 20$

The perimeter is 20 m.

b. $P = 2\ell + 2w$
$= 2 \times 4 + 2 \times 4$
$= 8 + 8$
$= 16$

The perimeter is 16 m.

c. $P = 2\ell + 2w$
$= 2 \times 6.5 + 2 \times 2.5$
$= 13 + 5$
$= 18$

The perimeter is 18 m.

2. a. $A = \ell w$
$= 7 \times 3$
$= 21$

The area is $21\ \text{m}^2$.

b. $A = \ell w$
$= 4 \times 4$
$= 16$

The area is $16\ \text{m}^2$.

c. $A = \ell w$
$= 6.5 \times 2.5$
$= 16.25$

The area is $16.25\ \text{m}^2$.

3. a. $V = \ell wh$
$= 7 \times 3 \times 2$
$= 42$

The volume is $42\ \text{m}^3$.

b. $V = \ell wh$
$= 4 \times 4 \times 4$
$= 64$

The volume is $64\ \text{m}^3$.

c. $V = \ell wh$
$= 6.5 \times 2.5 \times 3$
$= 48.75$

The volume is $48.75\ \text{m}^3$.

4. a. $C = \pi d$

$\doteq 3.14 \times 7$

$\doteq 21.98$

The circumference is about 21.98 cm.

b. $C = \pi d$

$\doteq 3.14 \times 12.5$

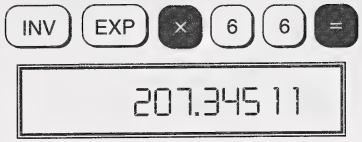
$\doteq 39.25$

The circumference is about 39.25 cm.

Enrichment

1. One complete rotation is the same distance as the circumference of the bicycle wheel.

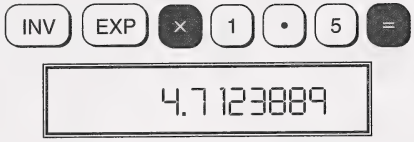
Use the formula $C = \pi d$.



The bicycle travels about 207 cm when the tire makes one rotation.

2. One rotation of the tractor tire is the same distance as the circumference of the tire.

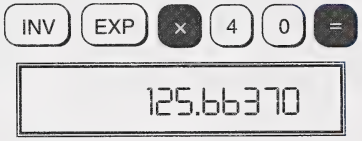
Use the formula $C = \pi d$.



The tractor travels about 4.7 m when the tire makes one rotation.

3. Step 1: Find the distance "travelled" in one wheel rotation.

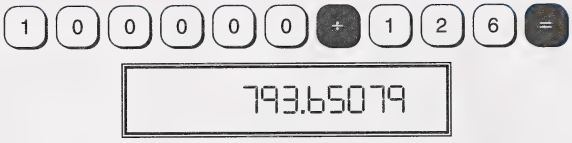
Use the formula $C = \pi d$.



The distance "travelled" in one wheel rotation is about 126 cm.

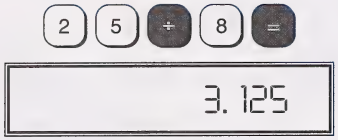
Step 2: Find the number of rotations in 1 km.

1 km = 100 000 cm

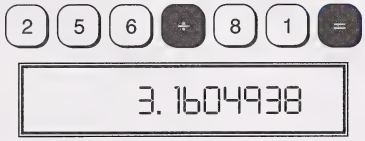


So, the wheel will turn about 794 times for the odometer to show a distance of 1 km.

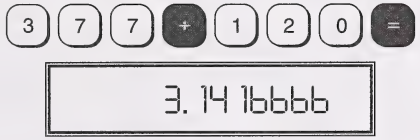
4. a.



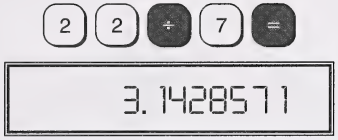
b.



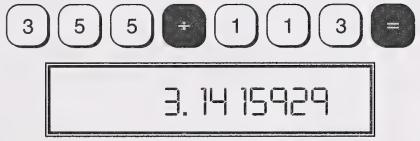
c.



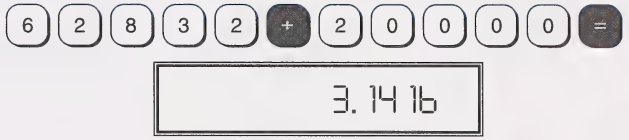
d.



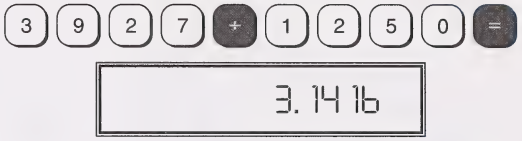
e.



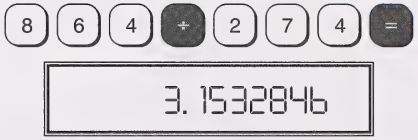
f.



g.



h.



5. a. The fraction $\frac{25}{8}$ is accurate to 1 decimal place.

3. 1 4 1 5 9 2 6

same same different

3. 1 2 5

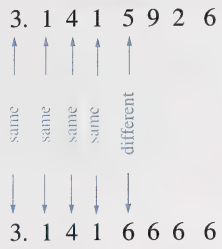
b. The fraction $\frac{256}{81}$ is accurate to 1 decimal place.

3. 1 4 1 5 9 2 6

same same different

3. 1 6 0 4 9 3 8

c. The fraction $\frac{377}{120}$ is accurate to 4 decimal places.

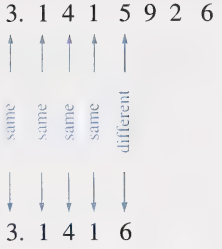


d. The fraction $\frac{22}{7}$ is accurate to 2 decimal places.

e. The fraction $\frac{355}{113}$ is accurate to 6 decimal places.

f. The fraction $\frac{62832}{20000}$ is accurate to 4 decimal places.

g. The fraction $\frac{3927}{1250}$ is accurate to 3 decimal places.

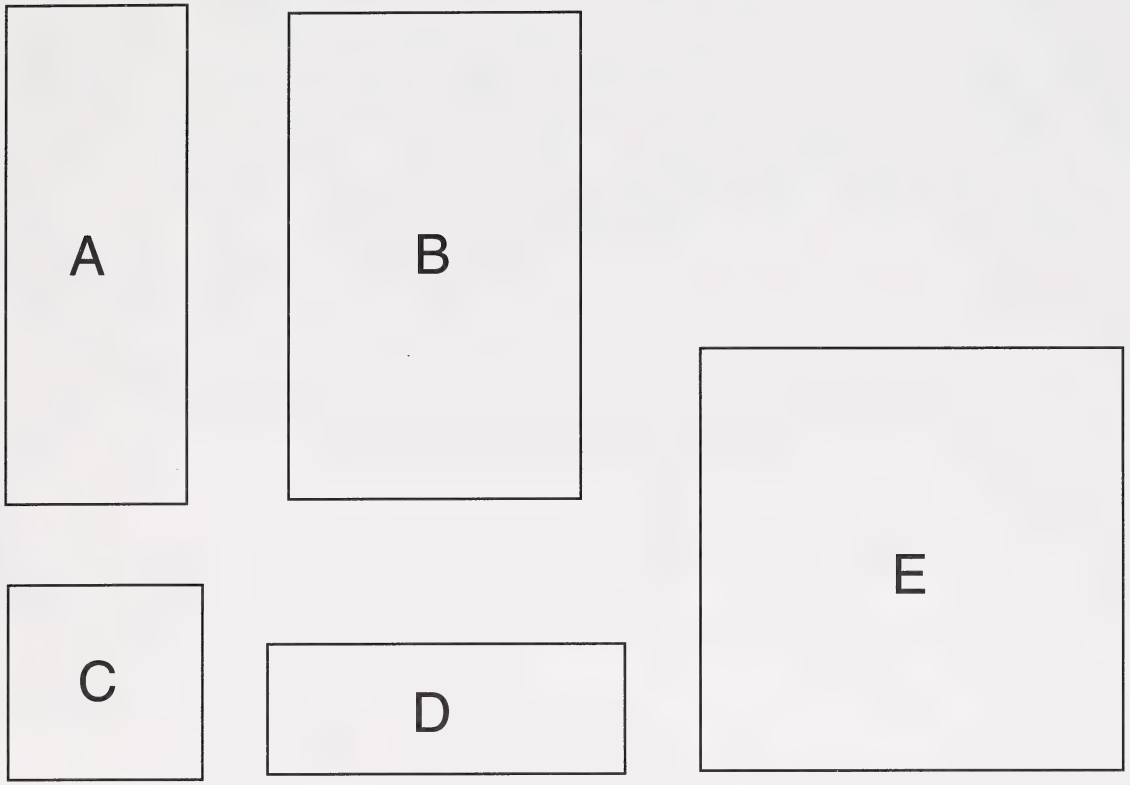


h. The fraction $\frac{864}{274}$ is accurate to 1 decimal place.

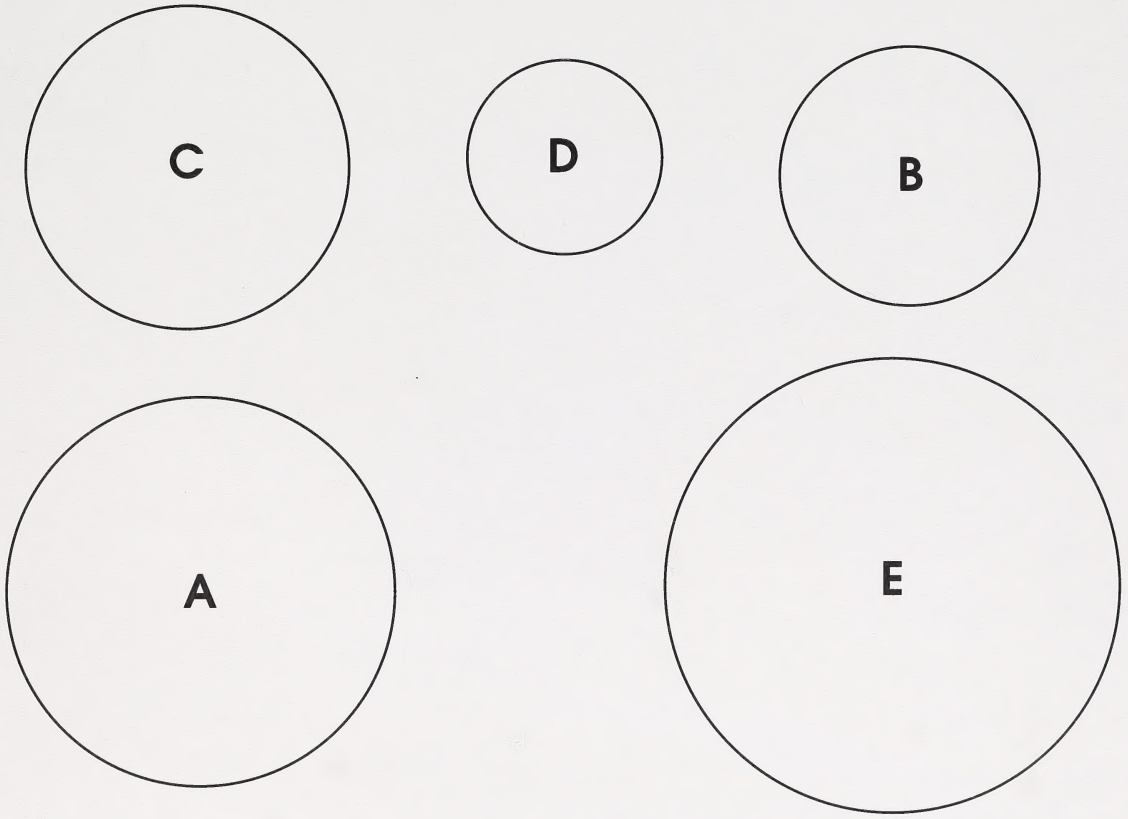
Number Tiles

0	5	10	15	
1	6	11	16	
2	7	12	17	20
3	8	13	18	
4	9	14	19	

Rectangles



Circles



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